FX Column: Fly High with Low Wings

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During a recent discussion with FX option traders the question came up of how relevant it really is to have a properly market-tuned extrapolation for low-delta volatilities in place, especially when vanilla options outside the 10-delta (or 90-delta) are not marked tradable on the electronic platform. Indeed, for price queries, which are then handled manually, the extrapolation scheme may not be relevant, but for portfolio valuation and risk management it is.

Our discussion quickly led to the fraud-scandal at WGZ-Bank's FX options trading team in 1998. WGZ eventually merged and was integrated into DZ-Bank. Tagesspiegel reported on 5 November 1998:[1]

> Two employees had embezzled DM 377 million in foreign exchange options transactions with proprietary trading with criminal energy, not harming any of the bank's business partners but only WGZ-Bank. The two experienced foreign exchange options traders, who had been employed by WGZ-Bank for seven and 14 years respectively, had rather tried to conceal an imbalance for which they were responsible. To this end, they had manipulated the bank's trading and settlement system since the second quarter of 1997. They had inserted some intermediate values into the volatility curve used in dollar options trading, which measures the fluctuations in exchange rates, in such a clever way that the manipulations went unnoticed for a long time during the daily checks by the trading administration.
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> The gap in the system was discovered on the night of October 23 and the damage was verified by October 26. One trader, who would have had a good reputation in the banking community due to his successful career in Germany and abroad, is regarded as the instigator. The other, according to the bank, as a follower. both traders have admitted to the manipulations. They were dismissed without notice. WGZ-Bank has filed charges against them with the public prosecutor's office. The introduction of a new, stricter control system, with which the risks of all trading products are recorded, as had been necessary for capital adequacy in accordance with the new Principle I since the beginning of October, proved fatal for the two traders. This "multi-dimensional system" could no longer be manipulated in this way, explained the bank's experts.

Let's now recap what happened technically. We go back in time to USD-DEM, remember?

USD-DEM Market

We consider the currency pair USD-DEM on 14 October 1998 with market data as in Table 1. I found only an at-the-money volatility and took some reasonable values for butterfly and risk reversal from my memory, just for the purpose of illustration.

Spot	1.6413	ATM volatility	11.4%

[1] https://www.tagesspiegel.de/wirtschaft/betrugsskandal-bei-wgz-bank-584674.html

DEM 3 M Money Market	3.57%	25-Delta Risk Reversal	+0.9%
3 M Forward	1.6341	25-Delta Butterfly	0.25%

Table 1: USD-DEM Market Data as of 14 October 1998; source: ICE Data Services / Bloomberg

Interpolation and Extrapolation

I consider two methods to construct the 3-months volatility curve based on the input points ATM-volatility and 25-Detla butterfly and risk reversal.

Method 1: parabolic interpolation (and extrapolation) on the delta-space: on the interval from 0 to 1 for forward call delta in the Black-Scholes model $\Delta = N(d_+)$ we construct the parabola

$$\sigma(\Delta) = ATM - 2RR \cdot (\Delta - 50\%) + 16BF \cdot (\Delta - 50\%)^2$$

based on the market input ATM volatility, 25-delta Risk Reversal (RR) and 25-delta Butterfly (BF). This method goes back to Malz and has been extended by Reiswich and Wystup. This parabola has fixed ends at the interval ends 0 and 1, and when converted to strike space, the same values stay. Consequently, on the strike space or log-moneyness space k=ln(K/F) (K denoting the strike price and F the forward price), the extrapolation in the wings flattens, see red smile curve in Figure 1.

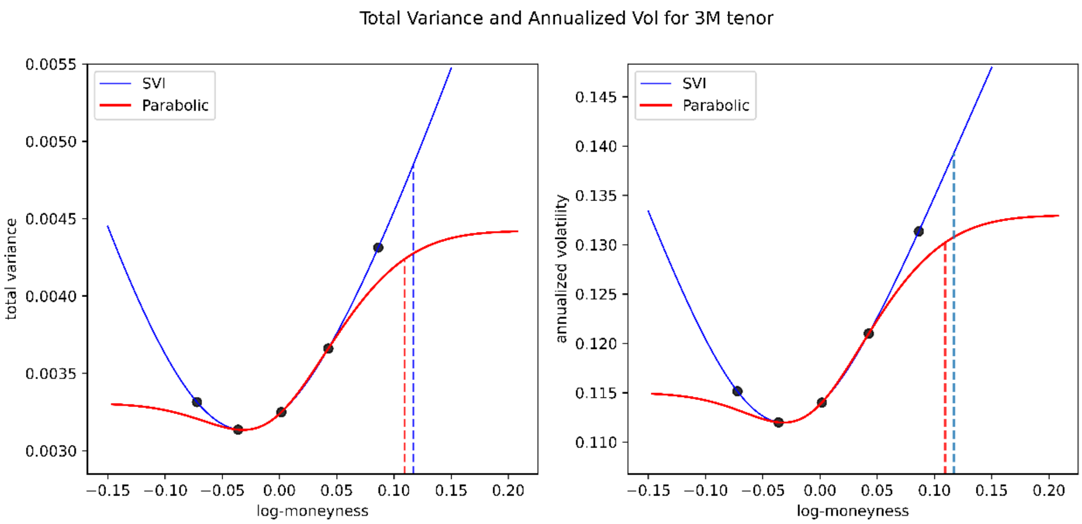

Figure 1: USD-DEM 3-Months Volatility Smile log-moneyness Space on 14 October 1998; total variance on the left, annualized volatility on the right.

Method 2: SVI fitting of the usual 5 input quantities ATM, 25-detla RR, 10-delta RR, 25-delta BF and 10-delta BF on the log-moneyness space k for the variance v(k):

$$v(k) = a + b\left(\rho(k-m) + \sqrt{(k-m)^2 + \sigma^2} \right)$$

The method goes back to Gatheral, has been extended and generalized by several authors and fits the 5 parameters a, b, ρ, m, σ to the input volatilities (black dots in Figure 1) using best fit optimization to generate the blue hyperbola in Figure 1.

We observe that inside the 25-delta range (inner three black dots) the interpolation of the two methods yields very close results. The big difference occurs on the wings. The Gatheral-hyperbola eventually grows linearly, whereas the Malz-parabola flattens on the wings by construction. It is known market practice that wing volatilities rather look like the hyperbola than a parabola, although the exact shape in the wings can only be verified by traded variance swap prices.

Sleight of Hand

Those wondering how I got the 10-detla quotes in 1998 would be right to guess that I used the extension equations 10BF = 3.7 × 25BF and 10RR = 1.8 × 25RR, the magic factors many market data providers use to fill in the blank and pretend that there is data. After all, tables and curves ought to look nice. This obviously leaves room for creativity. Different factors or different 10-delta input quotes generate different volatility curves.

Impact on the Options Portfolio

Now that we know how to technically lower the volatility in the wings, this knowledge may act as a drug. Suppose you are short 50,000 low-delta vanilla options and you wish to show a much higher portfolio value than reality, then you want a less negative value, thus a smaller volatility. You may be really bold and use at-the-money (ATM) volatility for pricing low-delta options, and in the 90s you might have gotten away with it, at least in the context of risk management. A more advanced way to lower wing-volatilities is by choosing an interpolation method like the Malz-parabola or to use lower input values in an SVI.

Example

Let's consider a 5-delta USD call – DEM put: on the SVI-curve, this corresponds to the log-moneyness k=0.1170, on the parabola to k=0.1092. The levels are different, because delta depends on volatility and volatilities are different in the two methods, see the dotted vertical lines in Figure 1. The corresponding volatility is 13.93% yielding an option price of 23 pips for strike K=1.8369. In the parabolic model this strike comes with a lower (!) volatility 13.08% yielding an option price of 16.5 pips. On a USD notional of 10M, the option prices are DEM 23,000 vs. DEM 16,500, the latter being only 72% of the former. The difference of DEM 6,500 yields a value difference of 325 million on 50,000 such contracts. This seems to indicate that volatility manipulation at WGZ's trading desk must have been even more creative.

Summary

1. Volatility smile construction is crucial for correct pricing and valuation of options portfolios.

2. Interpolation methods mostly differ in the wings, i.e. when interpolation schemes are used for extrapolation.
3. Trading with different market makers still shows differences in option prices, even for vanilla options, based on their proprietary volatility interpolation methods. SVI is one common method in the market, but not the only one.

References

(1) Uwe Wystup: FX Options and Structured Products, Second Edition, Wiley 2017.

(2) Dimitri Reiswich and Uwe Wystup: FX Volatility Smile Construction Wilmott, Volume 2012, Issue 60, pages 58-69

(3) James Gatheral: A parsimonious arbitrage-free implied volatility parameterization with application to the valuation of volatility derivatives, Global Derivatives & Risk Management Conference 2004, Madrid.

(4) Allan M. Malz: Option-Implied Probability Distributions and Currency Excess Returns, SSRN eLibrary, 1979

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