

FX Column: Why Digitals don't Have all Percentage Price Quotes

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Many people ask me why in digitals they see a percentage quote in EUR, but no such quote in USD (or the other way round). I will explain the reason in today's FX column followed by a case study about a structured forward, the Forward Extra for a EUR-zone based treasurer selling USD, where a digital put serves as a building block.

EUR-USD Market

We consider the currency pair EUR-USD on 23 April 2025 with market data as in [Table 1](#).

Spot	1.1000	ATM volatility	8.858%
EUR 6 M Money Market	1.893%	25-Delta Risk Reversal	+0.863%
JPY 6 M Money Market	4.145%	25-Delta Butterfly	0.356%
6 M Forward	1.1126	ATM bid-offer in volatility	0.15%

Table 1: EUR-USD Market Data as of 23 April 2025; source: ICE Data Services

Spot had already gone up further, but for the purpose of illustration, I freeze the spot reference to 1.1000. Two interesting observations: (1) The risk reversal is positive which means that the USD is viewed as the riskier currency; and (2) butterfly has practically doubled compared the recent past before the change of government in the U.S. EUR-USD volatility has increased and is skewed up, illustrated in [Figure 1](#).

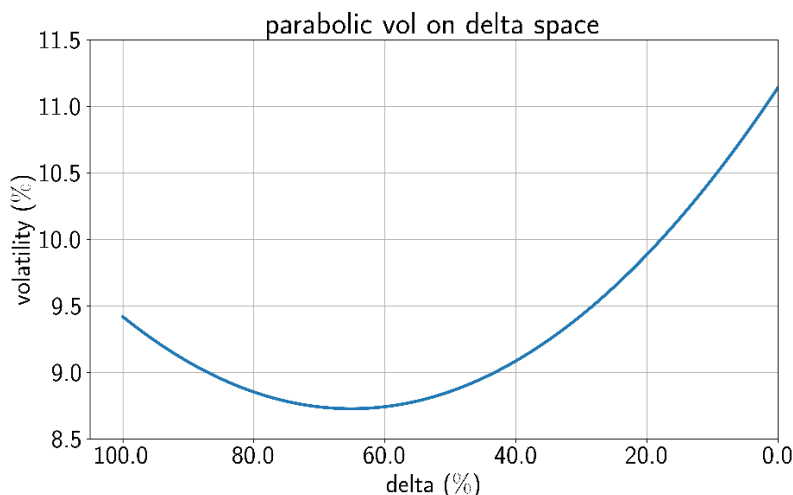


Figure 1: EUR-USD 6-Months Volatility Smile on (Call) Delta Space on 23 April 2025.

The corresponding volatility smile on the strike space for the spot in 6 months is shown in

[Figure 2](#). The slope of the smile on the strike space is a driving factor for the windmill-adjustment of digitals as explained in reference (3). In this market we would expect prices of digital puts with strikes around 1.0700 to be priced lower than in the Black-Scholes model.

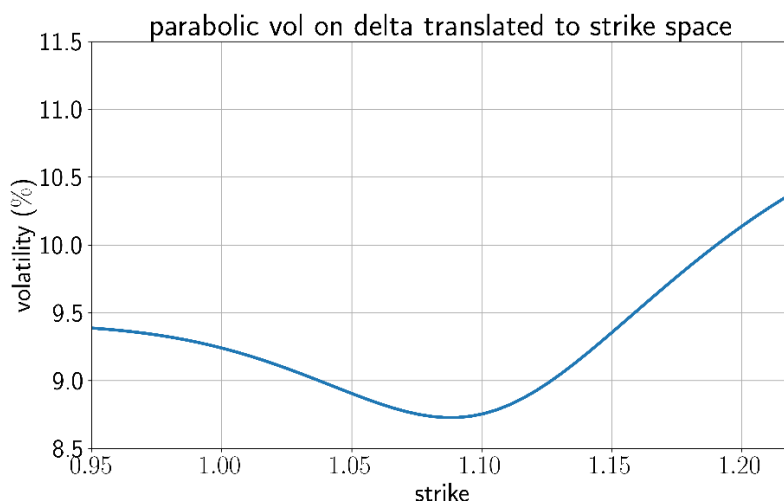


Figure 2: EUR-USD 6-Months Volatility Smile on Strike Space on 23 April 2025.

EUR Notional vs. USD Notional

Digital contracts in currency markets pay a fixed amount if the final reference spot is above the barrier (digital call) or below the barrier (digital put). The amount can be in either currency, *domestic currency* (USD here) or *foreign currency* (EUR here). In equity derivatives these contracts are often referred to as *cash-or-nothing* (domestic) or *asset-or-nothing* (foreign) digitals or binaries. Examples for both are shown in Figure 3.

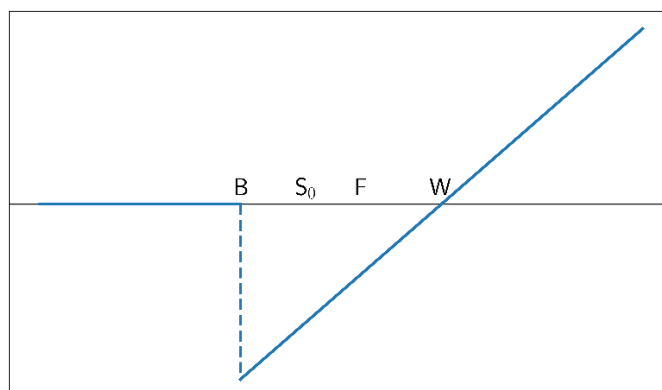


Figure 3: EUR-USD Digital Pricing in ICE Data (SuperDerivatives). LHS: DOM=USD Notional, RHS: FOR=EUR Notional.

We confirm the market price of 20.96% of the digital put with EUR payout to be lower than the Black-Scholes price of 22.61%, which is caused by the windmill-effect. Prices are shown in %EUR, i.e. in percentage of the EUR notional (=payout). The question many users of various platforms is often why we don't see the percentage price for the other digital, the one on the left-hand side with payout of USD 1 Million. In the screenshot, 0.0000 is shown, but that is obviously not the correct price.

The answer is simple. **There is no reasonable way to quote the price of a digital put with a payout in USD in percentage of a EUR notional, because this digital does not have a EUR notional.** For *vanilla* options, notionals can be converted with the strike, so the EUR 1M payout would be equivalent to USD 1.0736M. For

digital contracts, the EUR payout can only be converted to an equivalent USD payout at maturity with the final spot at maturity as a conversion factor, which is not known at trade time.

This might seem contradictory at first glance as the percentage prices of digitals can be seen as the discounted exercise probabilities $N(\phi d_+)$ for the EUR-paying digital ($\phi=+1$ for the call, $\phi=-1$ for the put) and $N(\phi d_-)$ for the DOM-paying digital. This interpretation is indeed correct, but, when calculating a percentage, one always needs to keep in mind to which amount the percentage is meant to be multiplied. And if we have a percentage for the USD case, but no USD amount to multiply with, then the quote does not make sense.

Similarly, if results were shown in USD, there would be a percentage quote for the digital put with the USD-payout, and no percentage price (indicated by 0.0000) for the digital put with the EUR-payout.

Prices in EUR or USD (not as percentage) can be calculated for both and are converted by the spot.

Case Study: Forward Extra for the USD-seller = EUR-buyer

An exporter in the EUR-zone is a USD seller = EUR buyer. A very common hedging strategy for the treasurer is a structured forward with a guaranteed worst case, called Forward Extra (or Forward Plus or Enhanced Forward), where buys a EUR call with strike at worst case W and sells a reverse-knock-in EUR put with strike at worst case W and lower knock-out barrier B , see [Figure 4](#).

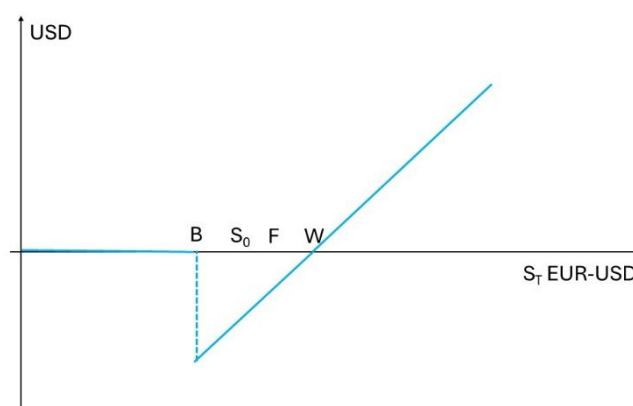


Figure 4: Payoff of a Forward Extra for the USD seller = EUR buyer: Long EUR call, short RKI EUR put.

In this example, the RKI is European, which means that the barrier is only valid at maturity. The worst case is usually chosen slightly worse than the market outright forward rate, and the barrier is solved so the bid price of the structure is zero for the treasurer.

Scenario Analysis:

- No knock-in event: treasurer is left only with a EUR call, and therefore, buys EUR at worst case if spot is above the worst case or at spot, if spot at maturity is lower (=better) than the worst case.
- Knock-in event: treasurer is long a (synthetic) forward contract and buys EUR at worst case.

Static Replication with Vanilla Options and a Digital:

For pricing with volatility smile effect, it is helpful to decompose the structure into basic building blocks with known market prices, i.e. vanilla options, whose prices follow directly from the smile curve, and a digital, which can be priced using the windmill-effect. In fact, the building blocks are:

- Long EUR call struck at W (no surprise);
- Short EUR put struck at B;
- Short digital put with barrier B and notional X.

Notional Currency of the Digital

My favorite question after decades working in currencies is “which currency?”, same question here.

If the notional for the underlying transaction is specified in EUR, i.e. the treasurer buys a fixed EUR amount and wants to *minimize the USD to spend* for this EUR amount, then the notional of the digital put is the difference of the worst case in USD and the best case in USD, i.e. $X = \text{EUR amount} \times (W - B)$, which is in USD, because W and B have the unit USD per EUR. The structure requires a digital put paying USD. Therefore, the price of the structure has no %EUR quote.

However, the more common setting for a EUR-zone based exporter is selling a fixed USD notional with the intention to *maximize the EUR amount to receive* when selling the USD. The notional of the digital put is now the difference between the best case in EUR and worst case in EUR, i.e. $X = \text{USD amount} \times (1/B - 1/W)$, which is in EUR. And that’s why we need a digital put paying EUR. Therefore, the price of the structure has no %USD quote.

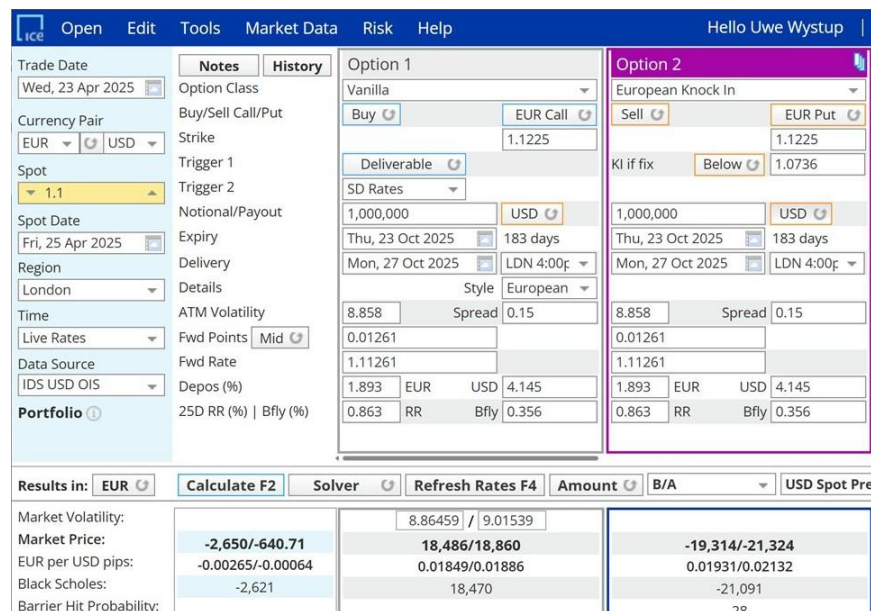


Figure 5: EUR-USD Forward Extra Replication; source: ICE Data Services (formerly known as SuperDerivatives).

Example

With the outright forward at 1.1126, we set the worst case 1 big figure higher, so $W = 1.1225$. The notional is USD 1 M. Solving for the barrier B yields 1.0736 in SuperDerivatives with a sales margin of EUR 640 and about EUR 1000 mid-offer spread for trading, see bid-offer quotes in Figure 5.

The Pricing of the Replication:

With a simple (educational) parabolic smile interpolator I get the building blocks and mid prices exhibited in

Table 2.

treasurer		strike	EUR notional	volatility	EUR premium	USD premium
buys	EUR call	1,1225	890.869	8,94%	18.639	20.503
sells	EUR put	1,0736	931.446	8,76%	-9.896	-10.886
sells	EUR dig put	1,0736	40.577		-10.556	-11.612
sum					-1.813	-1.995

Table 2: Pricing Example of a Forward Extra Replication in EUR-USD

- EUR notionals are USD 1M divided by the respective strikes.
- Notional of the digital put is $\text{USD } 1\text{M} \times (1/1.0736 - 1/1.1225)$.
- Volatilities are derived from the smile curve in
- [Figure 2](#).
- The price of the digital put is windmill-adjusted, see reference (4). Unadjusted TV with smile volatility would be EUR 10,963. The windmill-adjustment for a parabolic smile interpolation can be derived in closed form, see reference (5).
- For the *trading* team, there could be a mid-offer spread of about EUR 1,000. This would leave EUR 813 as sales margin and zero cost for the treasurer.

Summary

1. Digitals come up as building blocks in structured products with both foreign (EUR) and domestic (USD) payout, simply because of underlying transactions.
2. A European style Forward Extra can quickly be priced with a volatility curve and windmill-adjustment; even a parabolic interpolation yields prices close to the market.
3. EUR-paying digitals don't allow %USD price quotations, USD-paying digitals don't allow %EUR price quotations.

References

- (1) Wystup: FX Options and Structured Products, Second Edition, Wiley 2017.
- (2) Weber A. and Wystup, U.: Pricing Formulae for Foreign Exchange Options, *Contribution to [Encyclopedia of Quantitative Finance](#)*, John Wiley & Sons Ltd. Chichester, UK. 2010. pp.1408-1418.
- (3) Wystup: FX Column "How Can a 50/50 Bet Have Odds of 1:2 Instead of 1:1?", Wilmott Magazine, Volume 2018, Issue 98, November 2018, pp. 34-35
- (4) Wystup: FX Column "OTC Currency Digital Contracts – Traded Prices vs. Platform/Model Price", Wilmott Magazine, Volume 2023, Issue 127, September 2023, pp. 24-27.
- (5) Wystup: FX Column "FX Greeks", Wilmott Magazine, Volume 2019, Issue 99, January 2019, pp. 16-19.