

## FX Derivatives: Model and Product Trends

Uwe Wystup  
MathFinance AG  
[uwe.wystup@mathfinance.com](mailto:uwe.wystup@mathfinance.com)

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  - Wystup/Traders' Rule of Thumb
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  - Design Issues
  - Consistency Issues
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  - FX-Linked Swaps in City-Treasury Departments and Family Offices
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Other refs [Shkolnikov, 2009], [Janek, 2011]

## Wystup/Traders' Rule of Thumb 2003

[Wystup, 2003], [Wystup, 2006]: compute the cost of the *overhedge* of risk reversals (RR) and butterflies (BF) to hedge vanna and volga of an option EXO.

$$VV\text{-value} = TV + p[\text{cost of vanna} + \text{cost of volga}] \quad (1)$$

with

$$\text{cost of vanna} = \frac{\text{vanna} \text{EXO}}{\text{vanna} \text{RR}} \times \text{OH RR} \quad (2)$$

$$\text{cost of volga} = \frac{\text{volga} \text{EXO}}{\text{volga} \text{BF}} \times \text{OH BF} \quad (3)$$

$$p = \text{no-touch probability or modifications} \quad (4)$$

$$\text{OH} = \text{overhedge} = \text{market price} - TV \quad (5)$$

## Castagna/Mercurio 2007

[Castagna and Mercurio, 2007], [Castagna and Mercurio, 2006]: portfolio of three calls hedging an option risk up to second order (in particular the vanna and volga of an option).

$$c(K, \sigma_K) = c(K, \sigma_{BS}) + \sum_{i=1}^3 x_i(K) [c(K_i, \sigma_i) - c(K_i, \sigma_{BS})] \quad (6)$$

with

$$\begin{aligned} x_1(K) &= \frac{\frac{\partial c(K, \sigma_{BS})}{\partial \sigma}}{\frac{\partial c(K_1, \sigma_{BS})}{\partial \sigma}} \ln \frac{K_2}{K} \ln \frac{K_3}{K} \\ x_2(K) &= \frac{\frac{\partial c(K, \sigma_{BS})}{\partial \sigma}}{\frac{\partial c(K_2, \sigma_{BS})}{\partial \sigma}} \ln \frac{K}{K_1} \ln \frac{K_3}{K} \\ x_3(K) &= \frac{\frac{\partial c(K, \sigma_{BS})}{\partial \sigma}}{\frac{\partial c(K_3, \sigma_{BS})}{\partial \sigma}} \ln \frac{K}{K_1} \ln \frac{K}{K_2} \end{aligned} \quad (7)$$

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- Hedge with BF and RR vs Hedge with 3 Vanillas, and if vanillas then which ones?

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- Not clear how to translate to Double-Touch products
- Which volatility to use for the touch probability: ATM, average of ATM and barrier vol, derived from equilibrium condition  $NT_{vv} = NT_{bs} + NT_{vv}^* \dots$

## Comparison: Heston-Local-VV EUR-USD OT Mustache

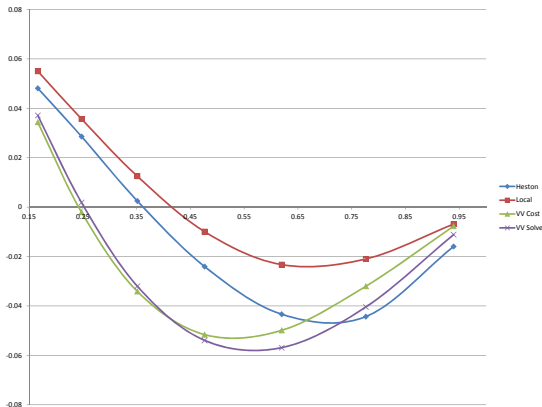


Figure: Model Comparison 6-M-EUR-USD-OT down paying USD: Market data of July 11 2012

## Comparison: Heston-Local-VV EUR-USD DNT Mustache

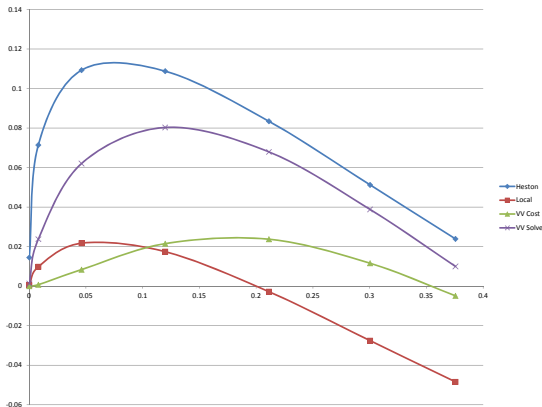


Figure: Model Comparison 6-M-EUR-USD-DNT paying USD: Market data of July 11 2012

## Comparison: Heston-Local-VV EUR-USD Down-Out-Call Mustache

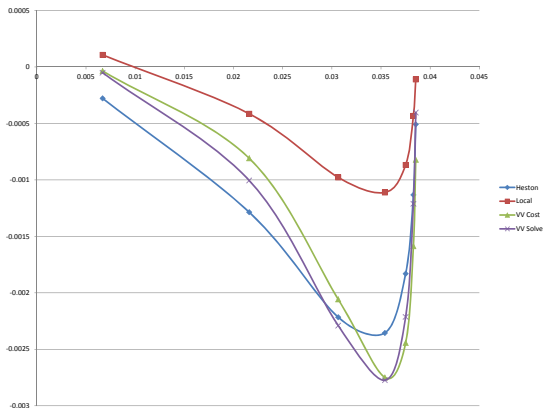


Figure: Model Comparison 6-M-EUR-USD down-out-call: Market data of July 11 2012



## Vanna-Volga Consistency Issues

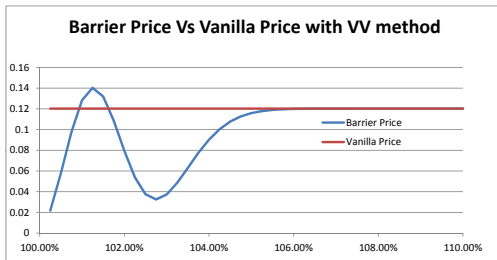
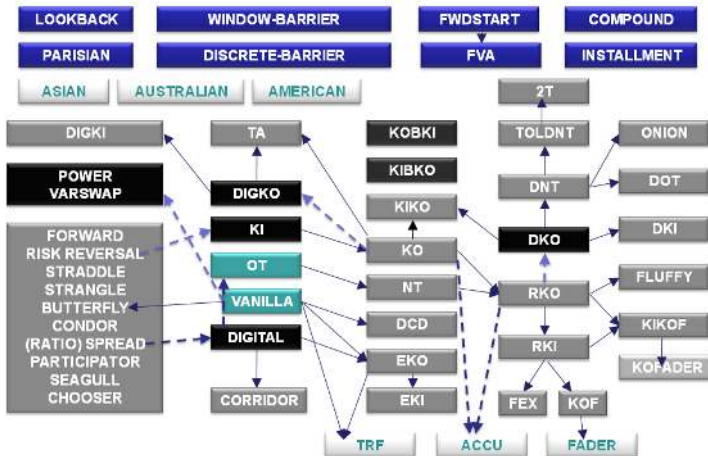


Figure: Convergence of a RKO EUR call CHF put to vanilla, strike 1.0809, 60 days, Market data of April 11 2012: Spot ref 1.20105, 2M EUR rate 0.055%, 2M-Forward -5.65, 10D BF 4.10, 25D BF 1.4755, ATM 3.00, 25D RR -0.7010, 10D RR -1.70.

## Vanna-Volga Consistency Issues



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$$\begin{aligned}dS/S &= r dt + \sigma(1 + \alpha(S - S_0) + \beta(S - S_0)^2)dW \\d\sigma &= \kappa(\theta - \sigma)dt + \epsilon\sigma dZ\end{aligned}$$

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- Several LSV models using different choices of local volatility functions in the equation for the spot or forward process combined with a stochastic process for the volatility, the variance, or a function of variance, have been considered and implemented in the following years in the industry. (See for one example [de Backer, 2006].)

## What Murex calls “Tremor”

Model <sup>1</sup> the forward as ( [Murex, 2008b], [Murex, 2008a])

$$df_t^T = \sqrt{v_t} \cdot g(f_t^T) dW_t^{(1)}, \quad (8)$$

$$dv_t = \kappa(\theta - v_t)dt + \eta\sqrt{v_t}dW_t^{(2)} \quad (9)$$

$$g(f) = \min(a + bf + cf^2, \text{cap} \cdot f) \quad (10)$$

$$\text{Cov}(dW_t^{(1)}, dW_t^{(2)}) = \rho dt \quad (11)$$

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<sup>1</sup>The name “Tremor” originates from Pascal Tremoreux, a senior member of the quant team at Murex

## What Murex calls “Tremor”

- $f_t^T$  forward to maturity  $T$  seen from  $t$
- $v_t$  variance process
- $v_0$  initial instantaneous variance
- $\theta$  long-term variance set to  $v_0$
- $\kappa > 0$  rate of mean-reversion
- $\rho$  instantaneous correlation between  $f_t^T$  and  $v_t$
- $\eta$  volatility of variance (vol of vol)
- $a, b, c$  parameters of  $g$  the *local volatility function*
- cap: 500%



## Jäckel/Kahl's Hyp Hyp Hooray

LSV model <sup>2</sup> introduced by Jäckel and Kahl in 2007 ([Jäckel and Kahl, 2010]), within equity options context, both of hyperbolic type in the parametric local vol and stochastic vol:

$$dx = \sigma_0 \cdot f(x) \cdot g(y) dW_t^{(1)}, \quad (12)$$

$$dy = -\kappa y dt + \alpha \sqrt{2\kappa} dW_t^{(2)} \quad (13)$$

$$f(x) = \left[ (1 - \beta + \beta^2) \cdot x + (\beta - 1) \cdot \left( \sqrt{x^2 + \beta^2(1 - x^2)} - \beta \right) \right] / \beta \quad (14)$$

$$g(y) = y + \sqrt{y^2 + 1} \quad (15)$$

$$\text{Cov}(dW_t^{(1)}, dW_t^{(2)}) = \rho dt \quad (16)$$

WLOG  $\beta > 0$ ,  $g(0) = 1$ ,  $x(0) = 1$ ,  $f(1) = 1$ .

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Pros:

- analytical approximation for implied volatilities

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[illegible]

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## Contained in PricingPartners: "Andersen-Hutchings"

[Andersen and Hutchings, 2008], a generalization of [Piterbarg, 2005], consider a normalized asset

$$dX_t = \lambda(t)\sqrt{V_t} \left( 1 - b(t) + b(t)X_t + \frac{1}{2}c(t)(X_t - 1)^2 \right) dW_t^{(1)}, \quad (17)$$

$$dV_t = \kappa(1 - V_t)dt + \eta(t)\sqrt{V_t}dW_t^{(2)}, \quad V_0 = 1, X_0 = 1 \quad (18)$$

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The model parameters are:

- $b(t)$  is the linear skew parameter, being a time-dependent function.
- $c(t)$  is the quadratic skew parameter, being a time-dependent function.
- $\lambda(t)$  is a time-dependent function.
- $\kappa$  is the mean reversion speed of volatility.
- $\eta(t)$  is the volatility of variance, being a time-dependent function

## "Silvano"

$$dS_t = (r_t^d - r_t^f)S_t dt + \sigma_t S_t dW_t^{(1)}, \quad (20)$$

$$dV_t = \eta_t (\ln z_t \theta_t - \ln V_t) V_t dt + \Sigma_t dW_t^{(2)}, \quad (21)$$

$$L_t = f(S_t, \Sigma_t, \rho_t, t), \quad (22)$$

$$\sigma_t = \min[\text{cap}, z_t(\omega_t V_t + (1 - \omega_t)L_t)], \quad (23)$$

$$\text{Cov}[dW_t^{(1)}, dW_t^{(2)}] = \rho_t dt, \quad (24)$$

where

## "Silvano"

$S_t$  : spot price (25)

$r_t^d$  : domestic rate (26)

$r_t^f$  : foreign rate (27)

$\sigma_t$  : volatility (28)

$V_t$  : stochastic volatility (29)

$\eta_t$  : mean reversion speed of stochastic volatility (30)

$\theta_t$  : equilibrium level of stochastic volatility (31)

$\Sigma_t$  : volatility of volatility (32)

$z_t$  : volatility drift factor (33)

$L_t$  : local volatility (34)

$\omega_t$  : stochastic volatility weight (35)

$\rho_t$  : spot-volatility correlation (36)

cap : volatility cap, currently set at 1000% (37)

Cost of Local Volatility Function (38)

## Pagliarani & Pascucci: SLV with Jumps

Risk-neutral dynamics for the log-price of the underlying asset [Pascucci and Pagliarani, 2012]

$$dX_t = \left( \bar{r} - \mu_1 - \frac{\sigma^2(t, X_{t-})v_t}{2} \right) S_t dt + \sigma(t, X_{t-})\sqrt{v_t}dW_t^{(1)} + dZ_t, \quad (39)$$

$$dv_t = k(\theta - v_t)V_t dt + \eta\sqrt{v_t} \left( \rho dW_t^{(1)} + \sqrt{1 - \rho^2}dW_t^{(2)} \right), \quad (40)$$



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- $\bar{r} = r_d - r_f - \int (e^y - 1 - y\mathbb{I}_{\{-1 < y < +1\}})\nu(dy)$
- $Z$  pure-jump Lévy process independent of the  $W$ , with triplet  $(\mu_1, 0, \nu)$
- Model is characterized by the local volatility function  $\sigma$ , the variance parameters: initial variance  $v_0$ , speed of mean reversion  $k$ , long-term variance  $\theta$ , vol-of-vol  $\eta$  and correlation  $\rho$ ; and the Lévy measure  $\nu$ .

Analytical approximation of the characteristic function.

## Bloomberg's SVL Model [Tataru et al., 2012]

$$\frac{dS_t}{S_t} = (r_t^d - r_t^f)dt + L(S_t, t)V_t dW_t^{(1)}, \quad (41)$$

$$dV_t = \kappa(\theta - V_t)dt + \xi V_t dW_t^{(2)}, \quad (42)$$

$$\text{Cov}[dW_t^{(1)}, dW_t^{(2)}] = \rho_t dt, \quad (43)$$

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where the local volatility  $L$  and stochastic volatility  $V$  are mixed using  $0 \leq \lambda \leq 1$  for

$$\xi = \lambda \xi_{\max}, \quad \rho = \lambda \rho_{\max}. \quad (44)$$

## Bloomberg Screen Shot: Local Stochastic Vol Parameters

Currency Markets Menu | EUR USD X RATE Currency | 100ML | Message

<Menu> to Close, 1<Go> to Update, 2<Go> to Reset to Default

89 Asset - 90 Actions - 92 Products - 93 View - 94 Data & Settings - Option Valuation  
 01 Calc - Solver (Premium) - Load - Save - Add to Portfolio - Trade - Split View  
 60 Deal 1 - 62 -  
 50 Pricing - 50 Fixings

Strategy 1  
 Local

Price date  
 Asset  
 Spot  
 Style  
 Direction  
 Equity  
 Indenture  
 Rate  
 Lower Local  
 Upper Local  
 Frequency  
 Number Fixings  
 First Expiry Date  
 Last Expiry Date  
 Nominal LIB  
 Notional LIB  
 Term (Cash)  
 Annual LIB  
 27 Target Reached  
 Minus  
 Vol

Stochastic Local Volatility Parameters

Date	Correlation	Vol of Vol	Mixing Fraction
1. Today 03/13/13	-0.2328	129.7712%	65.44%
2. 2W 03/27/13	-0.2136	92.4341%	65.63%
3. 1M 04/12/13	0.1960	58.6827%	55.94%
4. 2M 05/13/13	0.1375	40.5628%	42.62%
5. 3M 06/12/13	-0.1275	44.5558%	42.39%
6. 6M 09/12/13	-0.1329	43.9697%	41.88%
7. 1Y 03/13/14	-0.1328	42.3554%	40.87%

1 Update 2 Reset Close

Premium EUR  
 Premium Date 03/15/13  
 Delta  
 Sticky Delta  
 Rho

50 Relative 1/Y Zoom 1 25%

452 BFW 5:16 EON's Fourth-Quarter Proprietary Trading Profit Plunges 89%  
 BN 5:16 \*CHINA LONGYUAN TO APPROVE FY RESULTS ON MARCH 25 :916 HK  
 451 IXS 5:16 [Delayed] Whitbread : Squeeze for organic, reported still firm  
 450 SSI 5:16 赣锋锂业 : 澄清公告  
 449 BSE 5:16 FUTURISTIC SEC (FPKG) Profit 0.05, 12/31/12

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FX Derivatives: Model and Products©by MathFinance AG 23 / 37

## FX Derivatives Product Trends: Target Forwards

- Investor sells EUR 5 big figures above current spot every day for one year.
- Terminates as soon as 30 big figures of profit have accumulated.
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  - ④ Pricing/Platforms

## Bloomberg Screen Shot: Target Redemption Forward (TARF)

The screenshot shows a Bloomberg terminal window with the following elements:

- Top Bar:** Currency Markets Menu, EUR-USD X-RATE Curren, OWL, Message, and various window icons.
- Navigation Bar:** <HELP> for explanation. P089, 1<Go> to select current style / strategy, <Menu> to close.
- Menu Bar:** 09 Asset, 90 Actions, 90 Products, 90 View, 90 Data & Settings, Option Valuation.
- Sub-Menu:** Strategies, Custom Strategies, Option Styles / Strategies.
- Search Panel:** A list of strategies with '62) Target Redemption Fwd (TARF)' selected. Other strategies include One touch (OT), Participating Fwd (PF), Range Forward (RF), Risk Reversal (RR), Seagull (SG), Sequential KIKOKIKI (KIKOSQ), Straddle (SD), Strangle (SN), Dual-Strike TARF (DSTARF), Knockin TARF (KITARF), Pivot TARF (PVTARF), Tracker Forward (TF), Vanilla American Style (AM), Vanilla European Style (EU), and Variance Forward (FE).
- Description Panel:** A detailed description of the Target Redemption Forward strategy, explaining its purpose to hedge currency at a better rate than the prevailing forward rate, its leveraged structure, and the conditions for final payment (full payment or capped payment).
- Buttons:** Select and Close buttons at the bottom of the strategy list.
- Bottom Bar:** A news ticker showing financial news, including ENEL's cost savings, Japan's tender for wheat, and Fitch's affirmation of Singapore's outlook.
- Footer:** FRACTAL GLOBAL MARKET VISUALIZATION, TRY OUR NEW APP... >>>APPS FRACTAL <GO>, uwe.wystup@mathfinance.com, FX Derivatives: Model and Products©by MathFinance AG, 25 / 37

## Bloomberg Screen Shot: OVML Pivot Target Forward

Currency Markets Menu | EUR-USD X RATE Currency | OVML

<HELP> for explanation.  
 Enter 1 <Go> to calculate

89 Asset - 90 Actions - 92 Products - 93 View - 94 Data & Settings - Option Valuation  
 0 Date Solver (Premium) Load Save Add to Portfolio Trade Split View

61 Deal 1 62 +  
 50 Pricing 50 Fixings

Strategy 1  
 Lot 1

Price data	04/15/13	05/02	None Market Data
Asset	EUR/USD		Points
Spot	1.2016		Forward
Strike	Client buys strategy		EUR Depn
Direction	02/12/14		USD Depn
Expiry	NY 10:00		Gamma
Delivery	1.2016		Vega
Rate	1.2005		Theta
Lower Level	1.1997		Rho
Upper Level			Advanced Greeks
Frequency	Monthly		Alpha
Number Periods	12		Beta
First Expiry Date	04/15/13		Gamma
Last Expiry Date	02/12/14		Sigma
Notional TTR	EUR	1,000,000.00	Notional Data
Notional GTR		2,000,000.00	Path
Tam. (Cash)	USD	250,000.00	Premium Std
Accum. TTR		0.00	
IF Target Reached			
Model	Geometric Premium		
Vol	MC Stochastic Vol		
	8.8/5%		

467 BSE 5:17 TAPARIA TOOLS (TPT) Profit 20.61, 12/31/12  
 466 TEL 5:17 G4S Reports 32pc Fall in Full-year Profits  
 465 BRN 5:17 CLOSING MALAYSIAN FOREIGN EXCHANGE: MARCH 13  
 464 IIS 5:17 NWS HOLDINGS <659> VOLUNTARY ANNOUNCEMENT  
 BN 5:16 \*ECONOMIST DENNIS GARTMAN COMMENTS ON CROPS IN E-MAILED REPORT

PRICE COMMODITY OPTIONS IN DIFFERENT CURRENCIES IN

uwe.wystup@mathfinance.com

## Pivot Target Forward P&L Scenarios

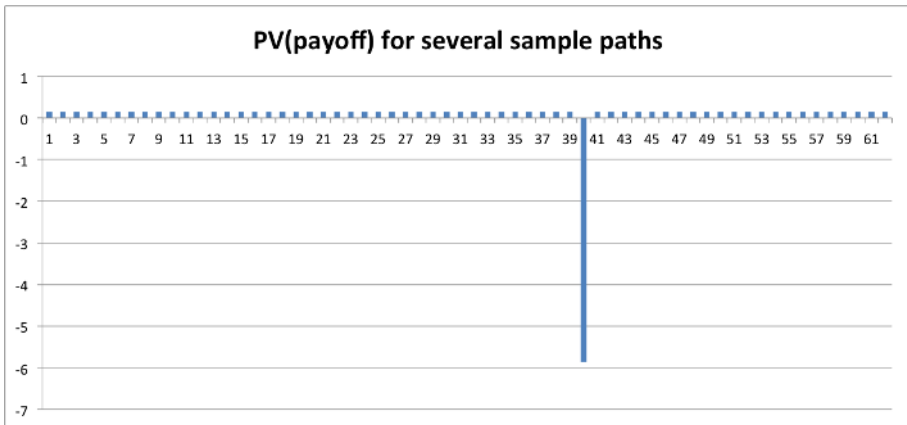


Figure: P&L Scenarios Pivot Target Forward

## Pivot Target Forward Payoff and Psychology

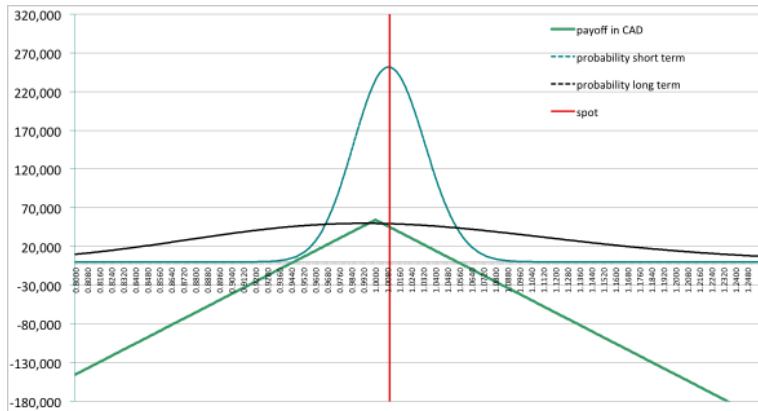


Figure: Pivot Target Forward Payoff and Psychology

## FX Derivatives in Digital Casinos



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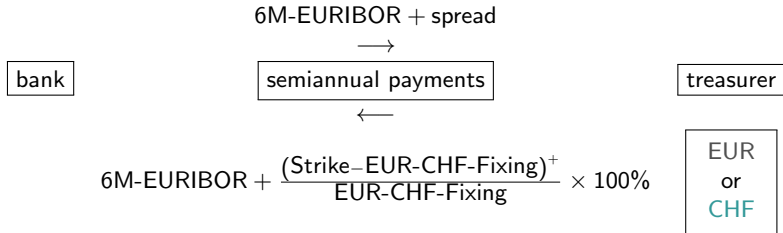
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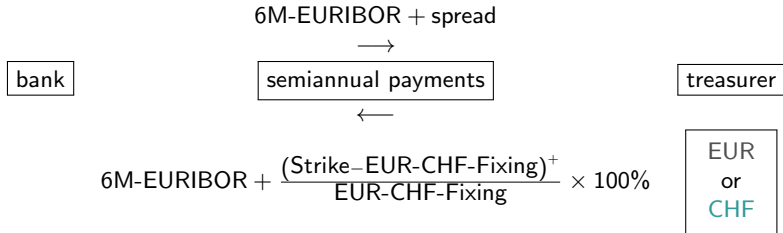
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- ③ geometric Brownian motion



## Currency Related Swap

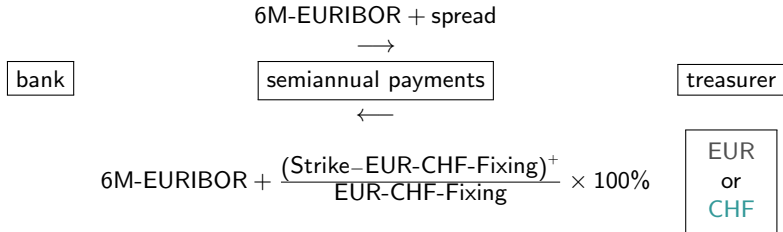


## Currency Related Swap



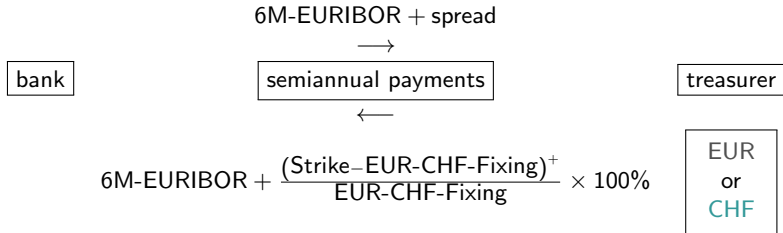
- Notional EUR 5M, 10Y swap

## Currency Related Swap



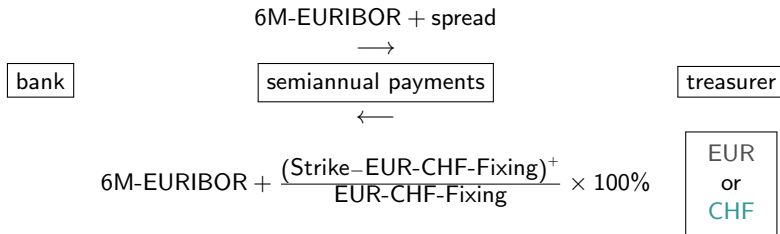
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- Strike 1.4350, spot 1.5960 on 5 Sept 2008, spread 2.09%

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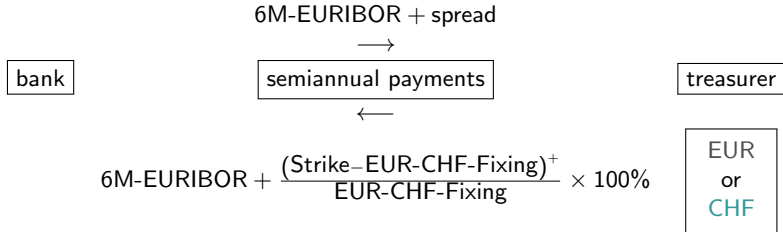
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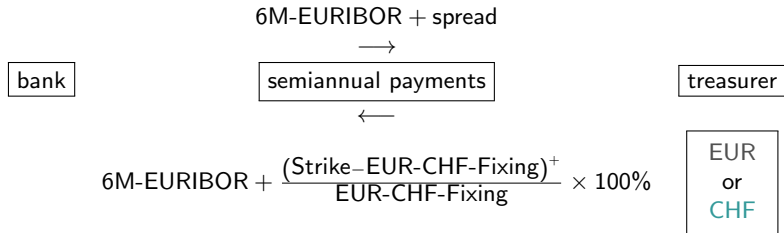
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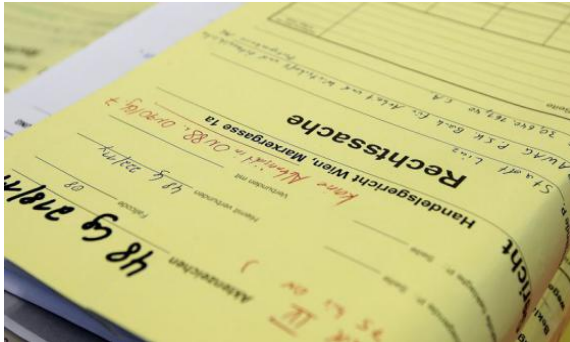
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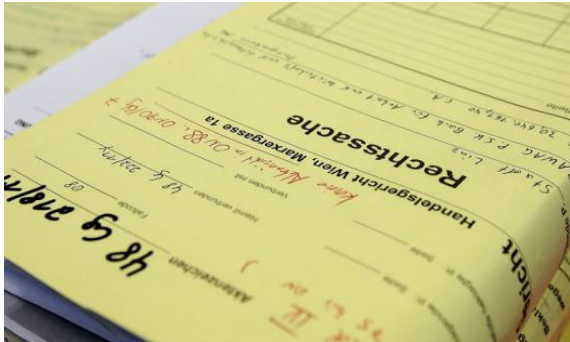
## Example: City of Linz ./ BAWAG



- In February 2007 Linz trades the 10Y CHF Swap 4175 containing a short sale of 20 self-quanto EUR-Put-CHF-Call options

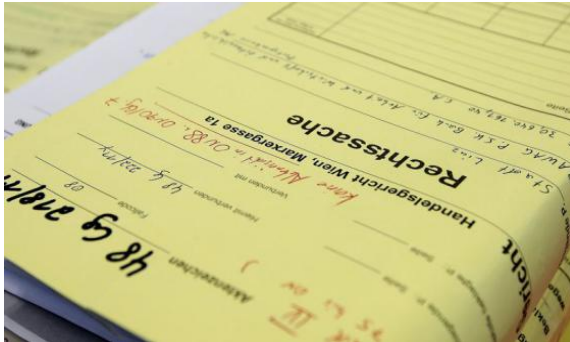


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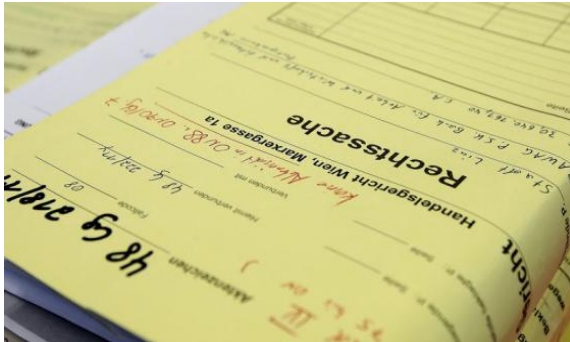
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- EUR-CHF *Carry Trades* lead to the *Linz-problem* for many treasurers/investors.

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- EUR-CHF *Carry Trades* lead to the *Linz-problem* for many treasurers/investors.
- CHF-loans done for the purpose of reducing interest rate payments require active position management.

## FX CFDs on Brokerage Platforms



- 1 Contract For the Difference (CFD) - Futures contract for retail investors

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## Comparison: DNT Mustache

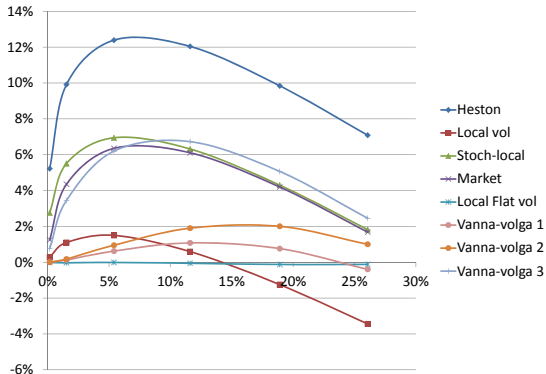


Figure: Model Comparison 6-M-EUR-USD-DNT paying USD: Spot ref 1.3068, 6M USD rate 0.11%, 6M-Forward 14.40, 10D BF 1.85, 25D BF0.45, ATM 11.99, 25D RR

## Summary

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- ③ Calibration of LSV models is the critical challenge.
- ④ Vanna-volga resurrects in digital casinos and private banking.

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