

# Numerical Methods for Derivatives

## Pricing

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## 1 Exercises

### 1.1 Find the strike for a given vanna

The second derivative of an option value function with respect to spot  $x$  and to volatility  $\sigma$  is often called *vanna*. From a trader's point of view vanna is responsible for the skew part of the volatility smile and hence a quantity often looked at in daily business. In order to offset a vanna position it is sometimes necessary to trade a vanilla option which produces a given vanna. The possible choices are contractual parameters such as time to maturity, strike and type (put or call). First convince yourself that vanna is independent of the type and does not vary a lot with different maturities. Consequently we will determine a strike which produces a given vanna. The value of a vanilla option in the Black-Scholes model is given by

$$v = e^{-r_d T} [FN(d_+) - KN(d_-)], \quad (1)$$

$$F = xe^{(r_d - r_f)T} \text{ "forward" }, \quad (2)$$

$$d_{\pm} \triangleq \frac{\ln \frac{F}{K} \pm \frac{1}{2} \sigma^2 T}{\sigma \sqrt{T}}. \quad (3)$$

View the vanna  $v_{x\sigma}$  as a function of the strike  $K$  and use Newton's method to retrieve the strike for a given vanna. Note that there can be either no, one or two solutions. The retriever should handle this problem by a suitable choice of the initial guess for the strike.

### 1.2 Wing Volatility Shift

Prove the theorem about the wing volatility shift of the section on volatility management.

### 1.3 Flaw in the Inversion Theorem

The Inversion Theorem by Gil-Palaez presented in class has a small weakness at the discontinuity points of the cumulative distribution function  $F$ . Formulate the problem and discuss a possible solution.

### 1.4 Interpretation of the Cross-Gamma/Correlation Risk relationship

Find an interpretation of the Cross-Gamma/Correlation Risk relationship.

### 1.5 Options on the minimum/maximum

Compute the value and Greeks of options on the minimum/maximum with minimal computational effort.

### 1.6 Details on Gauss-Integration

Search in the library to find out the details about how to compute weights and abscissas of the Gauss-Integration.

### 1.7 American style spread options

Use a two-dimensional binomial tree to compute the value of American style spread options.

## References

- [1] FLANNERY, B., PRESS, W., TEUKOLSKY, S. and VETTERLING, W. (1992). *Numerical Recipes in C*. Cambridge University Press.
- [2] SHREVE, S.E. (1996). *Stochastic Calculus and Finance*. Lecture notes, Carnegie Mellon University
- [3] WYSTUP, U. (2000). <http://www.mathfinance.de>