

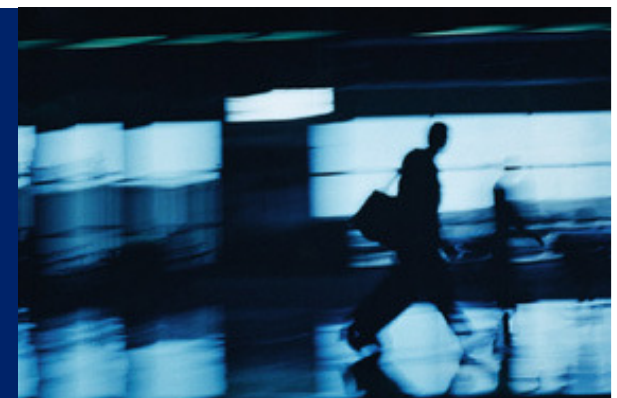
Commodity Derivatives

Modelling and Pricing in Practice

Dr. Tilman Huhne, Dr. Yuri Ivanov

MathFinance Conference

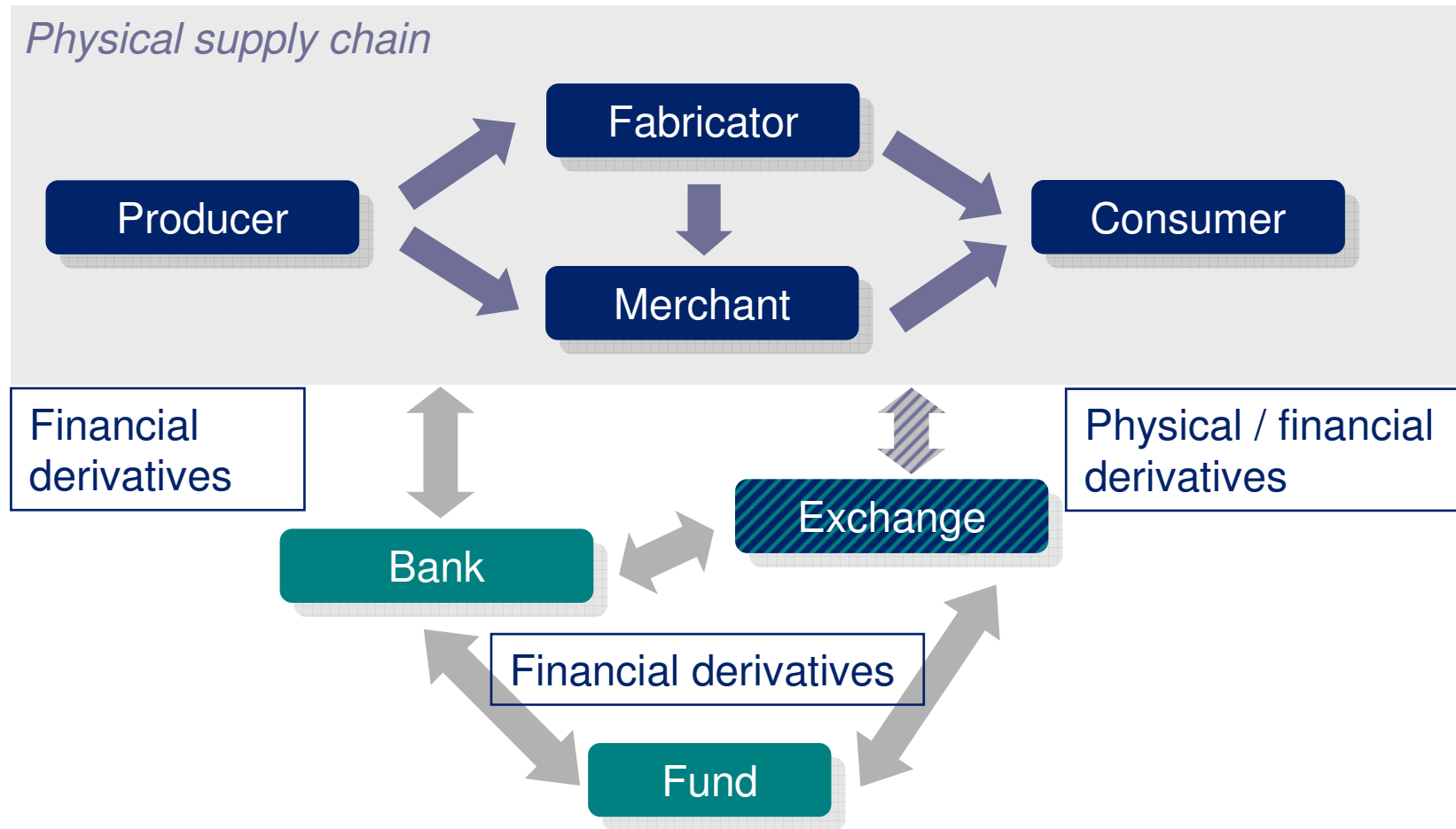
Frankfurt, March 23, 2009



Agenda

- Physical and financial Commodity Markets
- Commodity Spot and Forward Price Drivers
- Forward Curve Structure and Dynamics
- Challenges for Modelling
- Popular Modelling Approaches
- Model Selection and Validation
- Results
- Bibliography

Physical and financial Commodity Markets



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Commodity Spot and Forward Price Drivers (Examples)

Term	Supply	Demand
Short	<ul style="list-style-type: none"> <input type="checkbox"/> Total stocks available <input type="checkbox"/> Availability / usage of production capacity <input type="checkbox"/> Availability / usage of transport capacity <input type="checkbox"/> Weather conditions (where applicable) 	<ul style="list-style-type: none"> <input type="checkbox"/> Announcement of events possibly disrupting supply (stockpiling) <input type="checkbox"/> Weather conditions <input type="checkbox"/> Speculative trading
Medium/Long	<ul style="list-style-type: none"> <input type="checkbox"/> Total reserves (including recycling where applicable) <input type="checkbox"/> Total production capacity 	<ul style="list-style-type: none"> <input type="checkbox"/> Consumption by mature and emerging economies <input type="checkbox"/> Substitution by technological advances <input type="checkbox"/> Institutional investment (commodity index trackers etc.)

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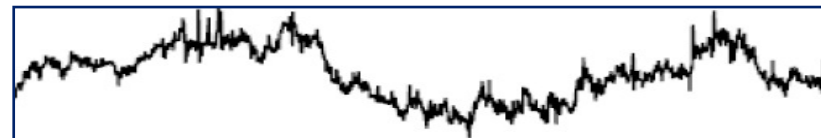
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Challenges for Modelling

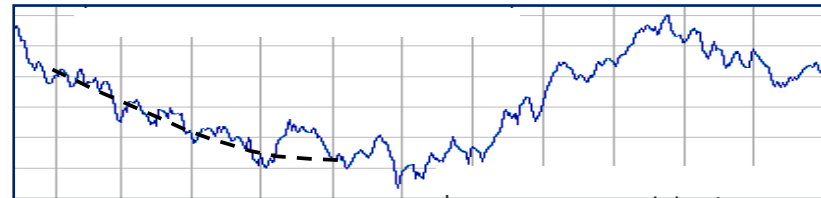
Spot price dynamics



Convenience yield dynamics

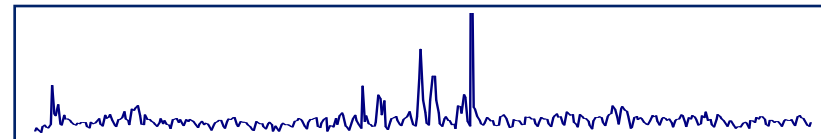


Mean reversion with fluctuating “mean”

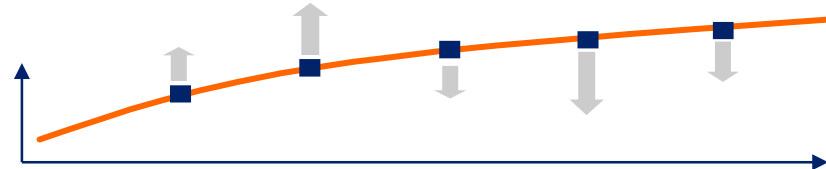


Seasonality patterns

Jumps and “spikes” (i.e. positive jump shortly followed by negative jump)



Correlated forward curve dynamics



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Popular Modelling Approaches

Schwartz-Smith two factor model [2]

$$\ln(S_t) = X_t + \xi_t$$

$$dX_t = (-\kappa X_t - \lambda_X)dt + \sigma_X dW_X$$

$$d\xi_t = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dW_\xi$$

$$dW_\xi dW_X = \rho dt$$

Mean reversion	✓
Seasonality	-
Stochastic vol	-
Jumps	-

S_t	Spot price at time t
X_t	Short-term deviation
ξ_t	Equilibrium level
κ	Mean reversion rate
μ_ξ	Equilibrium drift
λ_X, λ_ξ	Risk premiums
σ_X, σ_ξ	Volatilities
W_X, W_ξ	Correlated Brownian Motions

Popular Modelling Approaches (II)

Schwartz-Smith three factor model

$$\ln(S_t) = \chi_t + \xi_t$$

$$d\chi_t = (-\kappa\chi_t - \lambda_\chi)dt + \sqrt{u}dW_\chi$$

$$d\xi_t = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dW_\xi$$

$$du = \alpha(\eta - u)dt + \varepsilon\sqrt{u}dW_u$$

Mean reversion	✓
Seasonality	-
Stochastic vol	✓
Jumps	-

S_t	Spot price at time t
χ_t	Short term deviation
ξ_t, η	Equilibrium levels
α, κ	Mean reversion rates
μ_ξ	Equilibrium drift
$\lambda_\chi, \lambda_\xi$	Risk premiums
$\varepsilon, \sigma_\xi, \upsilon$	Volatilities and variance
$W_\chi, W_\xi, W_\upsilon$	Correlated Brownian Motions

Popular Modelling Approaches (III)

Clewlow-Strickland models [3,4]

$$dF_t^T / F_t^T = \sigma_1 e^{-\alpha(T-t)} dW_1 + \sigma_2 dW_2$$

One factor model

Two factor model

F_t^T	Forward price with maturity T at time t
α	Mean reversion rate
σ_1, σ_2	Volatilities
W_1, W_2	Correlated Brownian Motions

Mean reversion	✓
Seasonality	-
Stochastic vol	-
Jumps	-

- Expected change in forward price is zero under risk neutral measure
- Negative exponential form for forward price volatilities is consistent with Markovian spot price process
- Second factor: Shape of vol term structure

Popular Modelling Approaches (IV)

Modelling spot price seasonality [5]

Multiplicative representation

$$S_t^s = S_t w_{i(t)}$$

$$\sum_{i=1}^M w_i = 1$$

Additive representation

$$S_t^s = S_t + f(t)$$

$$f(t) = bt + \sum_{i=1}^N \left[\begin{array}{l} \beta_i \sin\left(\frac{2\pi it}{P}\right) \\ + \eta_i \cos\left(\frac{2\pi it}{P}\right) \end{array} \right]$$

S_t^s	Seasonal spot price at time t
S_t	Spot price at time t, corrected for seasonal effects
w	Seasonal weights
i(t)	Season corresponding to time t
M	Number of seasons
f(t)	Seasonal structure
b	Seasonal trend
P	Period length
β, η	Weights
N	Number of periodic terms

Popular Modelling Approaches (V)

Clelow-Strickland model with seasonal volatility and jumps

$$dF_t^T / F_t^T = \sigma_{seas}^T e^{-\kappa(T-t)} dW_1 + \sigma_2 dW_2 - \lambda \left[e^{e^{-\kappa(T-t)} \mu_J + \frac{1}{2} e^{-2\kappa(T-t)} \sigma_J^2} - 1 \right] dt + \left[e^{\omega_t^T} - 1 \right] dJ$$

Mean reversion	✓
Seasonality	✓
Stochastic vol	-
Jumps	✓

F_t^T	Forward price with maturity T at time t
σ_{seas}^T	Seasonal diffusion volatility (monthly piecewise-constant)
σ_2	Diffusion volatility
κ	Mean reversion rate
λ	Jump intensity
dJ	Poisson jump component
ω_t^T	Normally distributed with mean $\mu_J e^{-\kappa(T-t)}$ and variance $\sigma_J^2 e^{-2\kappa(T-t)}$
W_1, W_2	Independent Brownian Motions

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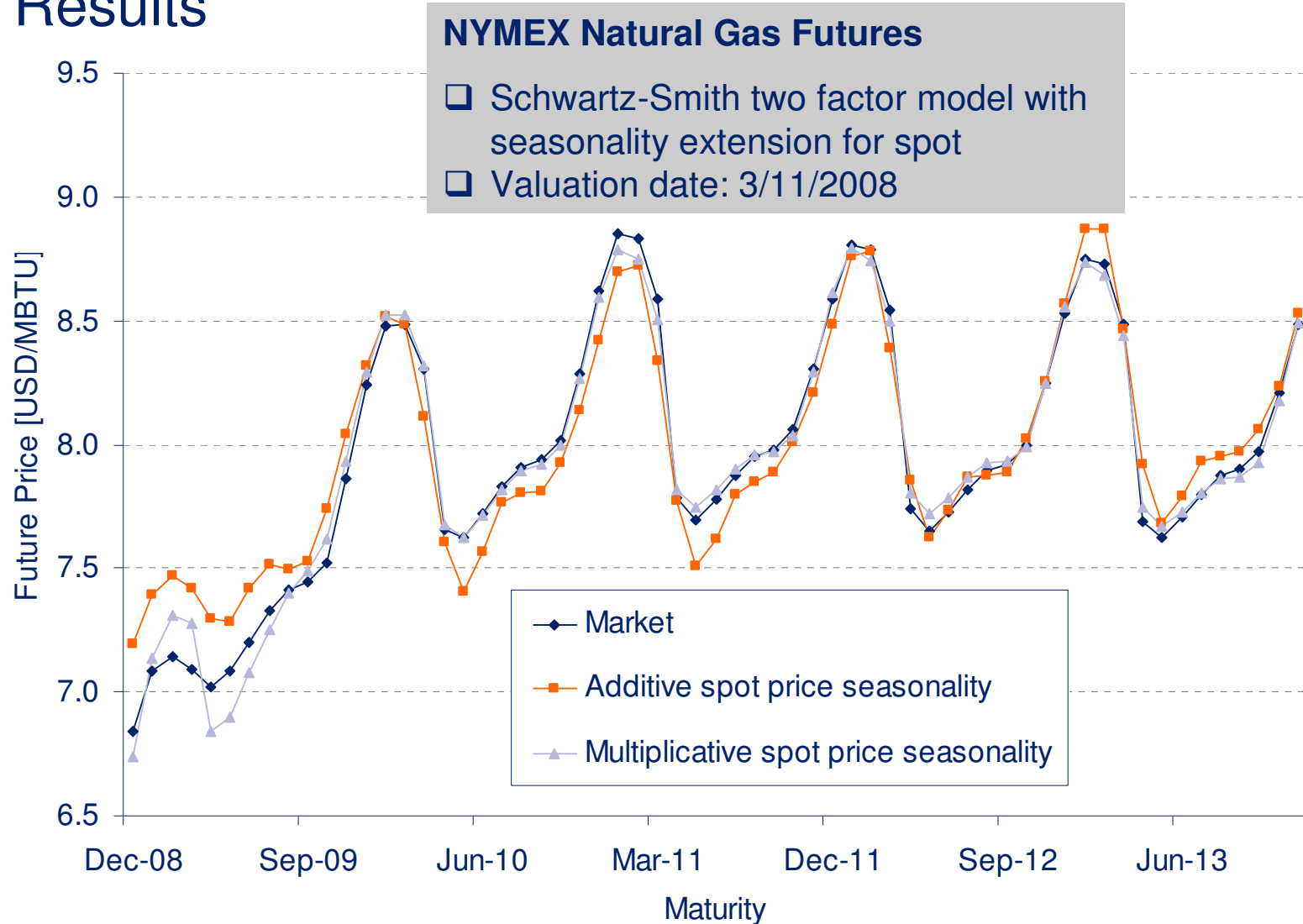
Model Selection and Validation

- ❑ Reproduction of empirical features of spot / forward price distributions
- ❑ Reproduction of observations in relevant derivative markets (volatility surface, exotic derivative prices etc.)
- ❑ Stability of estimated parameters (dependence on sample chosen for calibration, behaviour in case of market changes)
- ❑ Implementation and maintenance effort
- ❑ Runtimes for calibration
- ❑ Robustness (applicability in case of structural market changes etc.)

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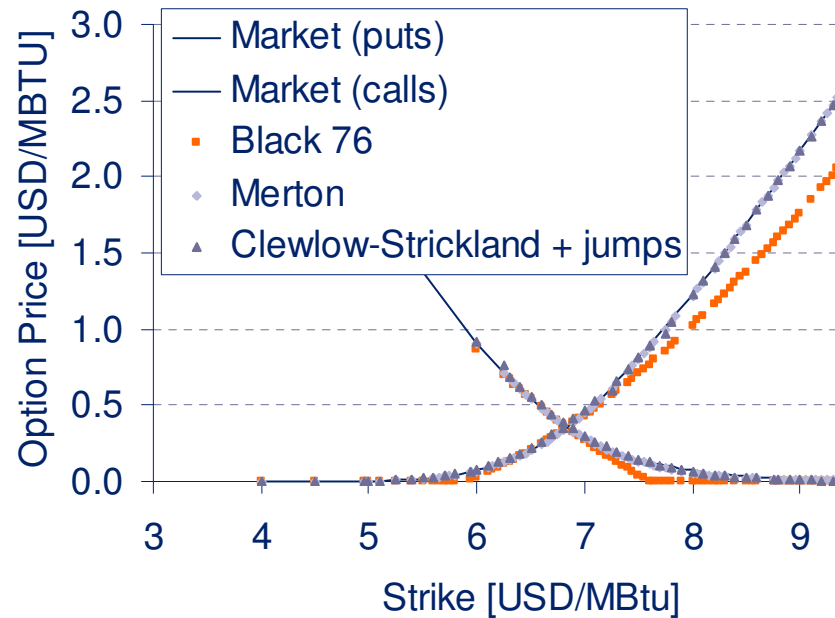
Results



Results (II)

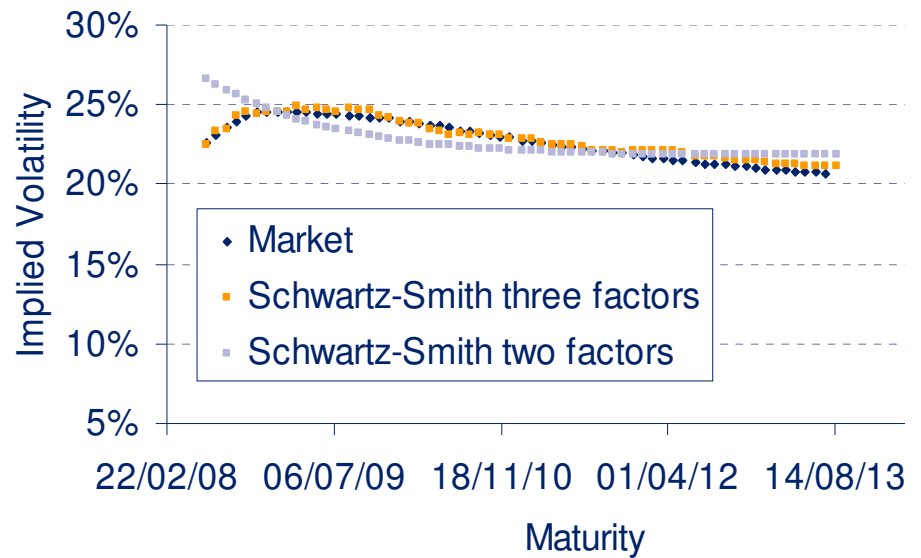
NYMEX Natural Gas Options

- ☐ Calibration: Futures, options
- ☐ Maturity date: 1/12/2008
- ☐ Valuation date: 3/11/2008



LME ATM Aluminium Calls

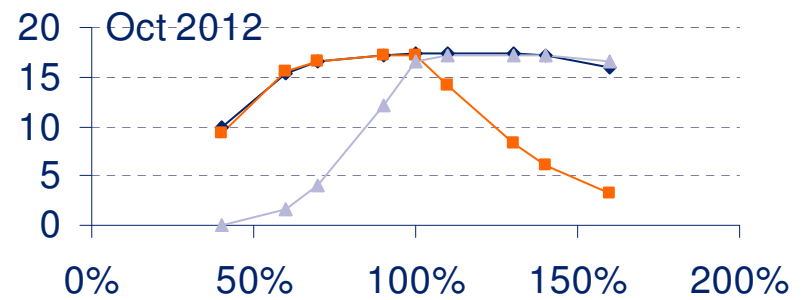
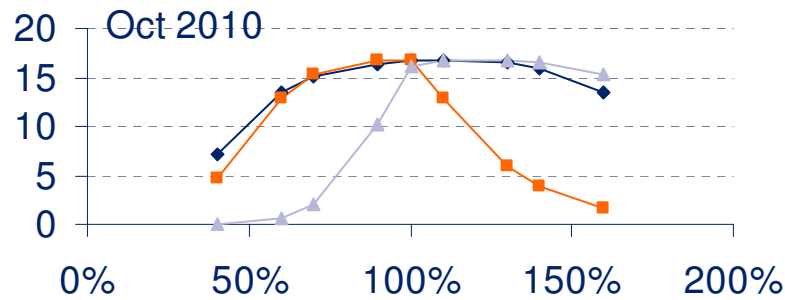
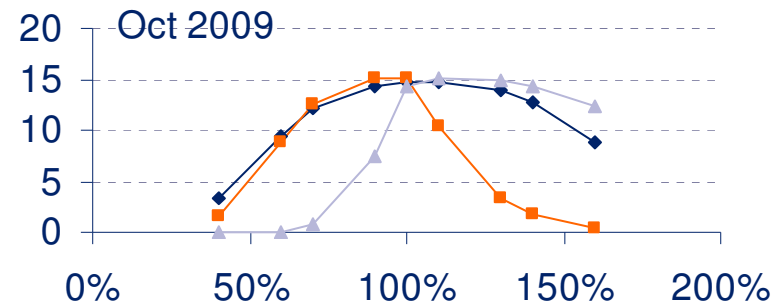
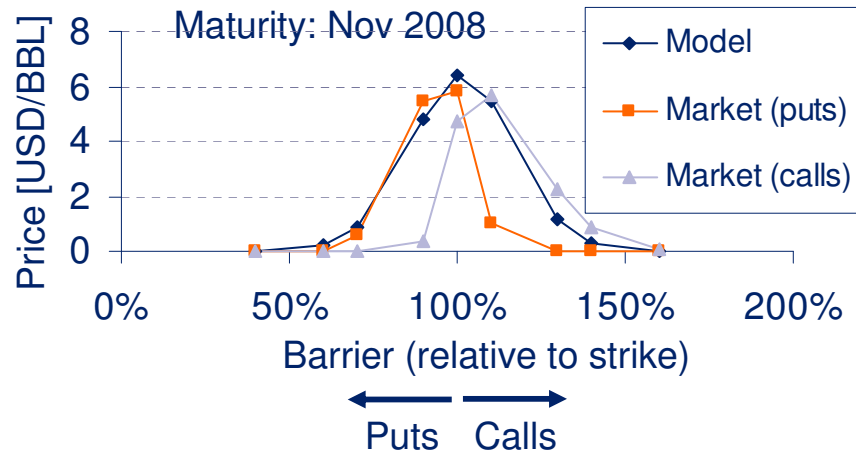
- ☐ Calibration: Futures, ATM options
- ☐ Valuation date: 13/6/2008



Results (III)

ICE ATM Crude Oil Knock-In Barrier Options

- Schwartz-Smith three factor model
- Calibration: Futures, ATM options
- Valuation date: 19/9/2008



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