Convexity Adjustments

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December 7, 2008

Executive Summary

This article aims to clarify the notion of *convexity* in fixed income markets. The main challenge is to provide a unified framework for all the different "convexity adjustments" that exist out there. We explain the basic and appealing idea behind the use of convexity adjustments and focus on the situations we believe are of particular importance to practitioners: yield convexity adjustments, forward versus futures convexity adjustments, timing and quanto convexity adjustments.

We claim that the appropriate way to look into any of these adjustments is as a side effect of a measure change, as proposed by Plesser (2003). When using the appropriate setup, there may be no immediate urge to do Taylor approximations or fall into too unrealistic assumptions. By using one unified framework, we hope to clarify some issues and help the reader realize that some of the assumptions that are sometimes imposed may be unnecessary.

For fixed income markets, *convexity* has emerged as an intriguing and challenging notion. Taking this effect into account correctly could provide financial institutions with a competitive advantage. The idea underlying the notion of a convexity adjustment is quite intuitive and can be easily explained in the following terms. Many fixed income products are non-standard with respect to aspects such as the timing, the currency or the rate of payment. This leads to complex pricing formulas, many of which are hard to obtain in closed-form. Examples of such products include in-arreas or in-advance products, quanto products, CMS products, or equity swaps, among others. Despite their non-standard features, these products are quite similar to plain vanilla ones whose price can either be directly obtained from the market or at least

computed in closed-form. Their complexity can be understood as introducing some sort of bias into the pricing of plain vanilla instruments. That is, we may decide to use the price of plain products and adjust it somehow to account for the complexity of non-standard products. This adjustment is what is known as convexity adjustment.

We start by classifying convexity adjustments into four classes :

- Yield Convexity Adjustments;
- Forward versus futures price adjustments;
- Modified schedule or timing adjustments; and
- Mismatch between currencies or quanto adjustments.

The yield convexity adjustment is somewhat unrelated to the remaining adjustments, but it is probably the "original one" in the sense that it is related to the non-linear (and convex) relationship between bond prices and their yield-to-maturity. The three remaining adjustments have traditionally been separated, both by practitioners and academics, as they concern different classes of products. Various ad hoc rules have been proposed in the literature to calculate a variety of convexity adjustments for different products. Many of them are, however, mutually inconsistent.

We start by critically analyzing the market practice for each of these types of adjustments. Then, we focus on timming adjustments and, in particular, on what we define to be *LIBOR adjustments* and *SWAP adjustments*. We show that LIBOR adjustments can be obtained in closed-form, up to the solution of a system of ODEs, in any affine term structure setting. Similar results can also be derived for SWAP adjustments provided we are willing to accept a (reasonable) assumption of the swap rate dynamics.

Previously existent results, such as the well-known results for lognormal LIBOR rates as in Pugachevsky (2001) or the Linear Swap Model (LSM) introduced by Hunt and Kennedy (2000) and further exploited by Hagan (2003) can be understood as particular cases of our, more general, results.

Key words: Convexity adjustment, LIBOR rate, Swap rates, in-arrears products, CMS, Forward price, futures price, forward martingale measure, swap martingale measure, affine term structures.

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