Optimization Software for Financial Mathematics

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Outline

Our Sources

One book and three websites

The book

Gerard Cornuejols and Reha Tütüncü,
Optimization Methods In Finance,
Cambridge University Press, Cambridge, UK ; New York, 2007

The websites

Decision Tree for Optimization Software (our share: 100%)
`http://plato.asu.edu/guide.html`

Benchmarks for Optimization Software (our share: 100%)
`http://plato.asu.edu/bench.html`

NEOS Server for Optimization (our share: 30%)
`http://neos.mcs.anl.gov`

The substructure in each area

Five parts

- The Problem: Mathematical Formulation
- Financial Mathematics Applications
- Commercial and "free" software
- interactive NEOS solvers
- Optimization Software Benchmarks

Outline

LP - Linear Programming

The Problem

Mathematical Formulation

objective $\min c^T x = \min \sum_{i=1}^{N} c_i * x_i$

variables $x_i, \quad i = 1, \ldots, N$

constraints $A * x = b, \quad x \geq 0$

LP - Linear Programming

FinMath Application I

Asset/Liability Cash Flow Matching (book 3.4)

- objective maximize wealth (at the end)
- variables assets, liabilities (monthly)
- constraints meet net cash flow

LP - Linear Programming

FinMath Application II

Asset Pricing and Arbitrage
Tax Clientele Effects in Bond Portfolio Management (book 4.4)

- goal construct optimal tax-specific bond portfolio for a given tax bracket exploiting price differential of after tax cash flows
- objective max (bid-price - asking-price)
- variables amounts of bonds
- constraints future cash flow nonnegative, bounded variables, adjustment for taxes

LP - Linear Programming
Software

Commercial and "free" packages/codes

- Many commercial and free codes solve LP
- **Commercial**: CPLEX, XPRESS-MP, LINDO, MOSEK, FortMP etc
- **Free**: BPMPD, PCx (IPM) - CLP, QSOpt, SOPLEX (Simplex) etc
- With modeling language: All commercial, BPMPD-AMPL, PCx-AMPL, BDMP-GAMS
- Sensitivity analysis: Best in commercial codes such as CPLEX

What is available at NEOS?

NEOS Server for Optimization http://neos.mcs.anl.gov/

Welcome to the NEOS server.

Our optimization solvers represent the state-of-the-art in optimization software. Optimization problems are solved automatically with minimal input from the user. Users only need a definition of the optimization problem; all additional information required by the optimization solver is determined automatically.

- User Feedback
- FAQ - NEOS Server
- Acknowledgements
- Collaborators

To submit your optimization job, first click on the **NEOS Solvers** icon to find a suitable solver.

LP solvers at NEOS

* BDMLP [GAMS Input]
* bpmpd [AMPL Input][LP Input][MPS Input][QPS Input]
* Clp [MPS Input]
* FortMP [MPS Input]
* MOSEK [AMPL Input][GAMS Input][MPS Input]
* OOQP [AMPL Input]
* PCx [AMPL Input][MPS Input]
* XpressMP [MOSEL Input][MPS Input]

Which benchmarks are available?

- Benchmark of commercial LP solvers (2-17-2008)

  CPLEX, MOSEK, LOQO, LIPSOL (Matlab)

- Benchmark of free LP solvers (8-7-2007)

  BPMPD, CLP, LPABO, LPAKO, QSOPT, SOPLEX, GLPK

- Large Network-LP Benchmark (commercial vs free) (2-5-2008)

  CPLEX, MOSEK, CLP, QSOPT

Outline

QP - Quadratic Programming

The Problem

Mathematical Formulation

objective $\min x^T Qx = \min \sum_{i,j=1}^{N} Q_{i,j} * x_i * x_j$ (convex)

variables $x_i, \quad i = 1, \ldots, N$

constraints $A * x = b, \quad x \geq 0$

QP - Quadratic Programming

FinMath Application I

Portfolio Selection, Asset Allocation (book 8.1)

- n number of securities
- x_i share invested in security i
- μ_i expected return of security i
- σ_i variance of security i
- $Q_{i,j} = \sigma_i * \sigma_j$
- goal make portfolio efficient

QP - Quadratic Programming
FinMath Application I (contd)

Portfolio Selection, Asset Allocation (book 8.1)

- either maximize return for given variance
- or minimize variance for given return R

objective $\min x^T Qx = \min \sum_{i,j=1}^N Q_{i,j} * x_i * x_j, \quad Q_{i,j} = \sigma_i * \sigma_j$

constraints $e^T x = 1, \quad \mu^T x \geq R, \quad x \geq 0, \quad e^T = (1, \ldots, 1)$

- short sales allowed if $x \geq 0$ is dropped

QP - Quadratic Programming

FinMath Application Ii

Further applications & formulations

- include transaction costs
- maximizing the Sharpe ratio (book 8.2)
- To reduce sensitivity of the QP
  - use robust optimization (below)
  - use Black-Litterman model (replace μ by product of distributions)

QP - Quadratic Programming
Software

Commercial and "free" packages/codes

- Several commercial and free codes solve QP
- **Commercial**: CPLEX, XPRESS-MP, LINDO, MOSEK, LOQO. FortMP etc
- **Free**: BPMPD, OOQP, GALAHAD, QPABO, QPS, CLP etc
- With modeling language: All commercial, BPMPD-AMPL, OOQP-AMPL,
- Sensitivity analysis: Best in commercial codes such as CPLEX

QP solvers at NEOS

* bpmpd [AMPL Input][LP Input][MPS Input][QPS Input]

* Clp [MPS Input]

* FortMP [MPS Input]

* MOSEK [AMPL Input][GAMS Input][MPS Input]

* OOQP [AMPL Input]

* XpressMP [MOSEL Input][MPS Input]

* All NLP solvers, two SDP solvers

Which benchmarks are available?

- Benchmark of commercial and other (QC)QP Solvers (2-5-2008)
  BPMPD, CPLEX, KNITRO, IPOPT, MOSEK, OOQP, QPB, LOQO
- AMPL-NLP Benchmark,
  IPOPT, KNITRO, LOQO, PENNLP, SNOPT (10-2-2007)

Outline

NLP - Nonlinear Programming

The Problem

Mathematical Formulation

objective $\min f(x)$ (possibly not convex)

variables $x_i, \quad i = 1, \ldots, N$

constraints $g_i(x) = 0, \quad i = 1, \ldots, m$

constraints $g_i(x) \geq 0, \quad i = m+1, \ldots, p$

NLP - Nonlinear Programming

FinMath Application I

Volatility Estimation (book 6.1)

- objective   optimize parameters in models to fit observed data (log-likelihood function)
- constraints   autoregressive time series, nonnegative residuals (GARCH)
- non-convex, unconstrained (recursive), nonlinearly constrained

NLP - Nonlinear Programming

FinMath Application II

Estimating Volatility Surface (book 6.2)

- mainly work by T. Coleman and coworkers
- Brownian motion model (for security movements)
- partial differential equations for European options
- estimate of volatility σ of underlying security needed
- leads to nonlinear least squares problems
- dependence on σ complicated; needs automatic differentiation

NLP - Nonlinear Programming

Software

Commercial and "free" packages/codes

- **Commercial**: KNITRO, SNOPT, LOQO, PENNLP, CONOPT, MOSEK (convex) etc
- **Free**: IPOPT, GALAHAD, HQP, SQPlab etc
- With modeling language: All commercial, IPOPT-AMPL, IPOPT-Matlab, SQPlab-Matlab

NLP solvers at NEOS

* CONOPT [GAMS Input]
* filter [AMPL Input]
* Ipopt [AMPL Input]
* KNITRO [AMPL Input][GAMS Input]
* LANCELOT [AMPL Input][SIF Input]
* LOQO [AMPL Input]
* MINOS [AMPL Input][GAMS Input]
* MOSEK [AMPL Input][GAMS Input]
* PATHNLP [GAMS Input]
* PENNON [AMPL Input]
* SNOPT [AMPL Input][FORTRAN Input][GAMS Input]

Which benchmarks are available?

- Benchmark of commercial and other (QC)QP Solvers (2-5-2008)
- AMPL-NLP Benchmark, IPOPT, KNITRO, LOQO, PENNLP, SNOPT (10-2-2007)

Outline

CO - Conic Optimization

The Problem

Mathematical Formulation

objective $\min c^T x = \min \sum_{i=1}^{N} c_i * x_i$

constraints $A * x = b, \quad x$ in cone C

SOCP cone $x_1^2 \geq x_1^2 + \ldots + x_N^2, \quad x_1 \geq 0$

SDP cone X symmetric positive definite, $X = vec(x)$

CO - Conic Optimization

FinMath Application SOCP

Tracking Errors (book 10.1)

- goal track difference between return of given portfolio and benchmark XDM (market index)

  the variance-constrained Markowitz QP becomes

- objective min $\mu^T(x - x_{XDM})$
- constraints $(x - x_{XDM})^T * \Sigma * (x - x_{XDM}) \leq TE^2$ (SOCP)
- constraints $A * x = b, \quad C * x \geq d$
- TE tracking error, Σ covariance matrix

CO - Conic Optimization

FinMath Application SDP

Approximating Covariance Matrix (book 10.2)

- Not estimating Covariance Matrix
- objective $\min ||\Sigma - \bar{\Sigma}||$, $\bar{\Sigma}$ in the SDP cone C
- constraints $\lambda_{\text{min}}(\bar{\Sigma}) \geq \delta > 0$

CO - Conic Optimization

FinMath Application

Further Applications

- Recovering risk-neutral probability from options prices (book 10.3)
- Arbitrage bounds for forward start options (book 10.4)

CO - Conic Optimization

Software

Commercial and "free" packages/codes

- SOCP
  - **Commercial**: CPLEX, MOSEK
  - **Free**: SDPT3, SeDuMi, YALMIP
  - With modeling language: All commercial, CVX, YALMIP (Matlab), CVXMOD (Python)
- SDP
  - **Free**: SDPT3, SeDuMi, YALMIP
  - With modeling language: CVX, YALMIP (Matlab), CVXMOD (Python)

Conic solvers at NEOS

* csdp [MATLAB BINARY Input][SPARSE SDPA Input]
* DSDP [SDPA Input]
* penbmi [MATLAB Input][MATLAB BINARY Input]
* pensdp [MATLAB BINARY Input][SPARSE SDPA Input]
* sdpa [MATLAB BINARY Input][SPARSE SDPA Input]
* sdpa-c [MATLAB BINARY Input][SPARSE SDPA Input]
* sdplr [MATLAB BINARY Input][SDPLR Input][SPARSE SDPA Input]
* sdpt3 [MATLAB BINARY Input][SPARSE SDPA Input]
* sedumi [MATLAB BINARY Input][SPARSE SDPA Input]

Which benchmarks are available?

- Several SDP-codes on SDP problems with free variables (7-23-2007)
- Several SDP codes on problems from SDPLIB (8-10-2007)
- SQL problems from the 7th DIMACS Challenge (8-8-2002)
- Newer SDP/SOCP-codes on the 7th DIMACS Challenge problems(10-24-2007)
- Several SDP codes on sparse and other SDP problems (8-8-2007)
- SOCP (second-order cone programming) Benchmark (10-28-2007)

Outline

RO - Robust Optimization

Any Problem with Data Uncertainty

Goal

Optimize for worst case scenario varying parameters in sets

constraint or model robustness

Solution must always be feasible (p uncertain)

original problem min $f(x)$ subj to $G(x, p)$ in K

robust problem min $f(x)$ subj to $G(x, p)$ in K for all p in U

objective or solution robustness

Solution must stay close to optimal (p uncertain)

original problem min $f(x, p)$

robust problem $\min_x \max_p f(x, p)$

RO - Robust Optimization

Examples, FinMath Application

Example I Uncertain LP

min $c^T x$ subject to $a^T x \leq b$

a,b varying in ellipses leads to SOCP

Example II Uncertain QCLP

min $c^T x$ subject to $x^T Qx + b^T x + c \leq 0$

Q,b,c varying in ellipses leads to SDP

Application: Robust Portfolio Selection (book 20.3)

Return μ and covariance Σ vary in intervals

$U = \{(\mu, \Sigma), \quad \mu^L \leq \mu \leq \mu^U, \quad \Sigma^L \leq \Sigma \leq \Sigma^U, \quad \Sigma \succeq 0\}$

Outline

DP - Dynamic Programming

The Problem

Ingredients

- decision stages
- decision states in each stage
- transitions from state to state
- value functions per state, best possible objective
- recursive relation between value functions

The Problem

- Decision Maker decides in each state -> policy or strategy
- Goal: achieve best overall objective
- deterministic DP if decisions define policy uniquely

DP - Dynamic Programming
Solution, FinMath Applications

Bellman's Principle

- All local decisions have to be optimal to achieve global optimality
- forward/backward recursion

Option Pricing (book 14.1/2)

- American options (random walk, binomial lattice models)

Structuring asset-backed securities (book 15.3)

- Collateralized mortgage obligation, CMO
- CMOs make cash flow more predictable by restructuring into bonds with different maturities

DP - Dynamic Programming

Software for DP

few general packages

- **Commercial**: P4 (Excel add-in)
- **Free**: OpenDP, DP2PN, RDP, CompEconTB

Outline

SP - Stochastic Programming

The Problem

Mathematical Formulation

- objective $\max a^T x + E(\max_y c(\omega)^T y(\omega))$
- ω random event, E expected value
- constraints $Ax = b, \quad B(\omega)x + C(\omega)y(\omega) = d(\omega)$
- constraints $x \geq 0, \quad y(\omega) \geq 0$

Classification

- This is a two-stage SP with recourse
- Analogously one defines multi-stage stochastic programs

SP - Stochastic Programming

FinMath Applications

Value-at-Risk, VaR, Conditional VaR

- CVaR: expected loss $\geq$ VaR (book 17.2)
- Example: Bond Portfolio Optimization (book 17.3)

Further applications

- Asset/Liability Management (book 18.1)
- Synthetic Options (book 18.2)
- Option Pricing with Transaction Costs (book 18.3)

SP - Stochastic Programming

Software

Commercial and "free" packages/codes

- **Commercial**: SLP-IOR (GAMS), SPInE, XPRESS-SP, FortSP etc
- **Free**: MSLiP, SMI (COIN-OR), BNBS

Stochastic solvers at NEOS

* bnbs [SMPS Input]

* ddsip [LP Input][MPS Input] (integer variables)

* FORTSP [SMPS Input]

* MSLiP [SMPS Input]

Outline

Mathematical Formulation

Any problem in which some variables have to attain integer values

Example I: MILP- mixed integer LP (book 12.1)

- **Combinatorial Auction**
- $M = \{i = 1, \ldots, m\}$ items to be auctioned
- $B_j = (S_j, P_j)$ bid, S_j subset of M, P_j price offer, $j = 1, \ldots, n$
- objective $\max \sum_j P_j x_j$ revenue
- constraints $\sum_j x_j \leq 1$, i in S_j, $i = 1, \ldots, m$
- constraints x_j in $\{0, 1\}$, $1/0$ if B_j wins/loses

Example II: MIQP - mixed integer QP (book 12.4)

- **Portfolio Optimization with Minimum Transaction Levels**
- Problem: investment x_j too small to be executed
- $\min x^T Qx$, subj to $\mu^T x \geq R, Ax = b, Cx \geq d$
- and if $x_j > 0$ then $x_j \geq l_j$ (min transaction level)
- use standard branch&bound (B&B): find j with $x_j < l_j$
- solve 2 QPs, one with $x_j = 0$ and one with $x_j \geq l_j$
- if not optimal, find next x_j, repeat
- Other applications: The lockbox problem (book 12.2), constructing an index fund (book 12.3)

IP - Integer Programming

Software

Codes for MILP, partly MIQP/MIQCQP

- **Commercial**: CPLEX, XPRESS-MP, LINDO, FortMP etc
- **Free**: SCIP, CBC, GLPK, MINTO, SYMPHONY, LP_SOLVE etc

Codes for MINLP

- **Commercial**: DICOPT, SBB, LINDO, BARON etc
- **Free**: BONMIN, FilMINT, MINLP, LaGO etc

Integer solvers at NEOS

* Cbc [AMPL Input][MPS Input]
* feaspump [AMPL Input][CPLEX Input][MPS Input]
* MINTO [AMPL Input]
* qsopt_ex [AMPL Input][LP Input][MPS Input]
* scip [AMPL Input][CPLEX Input][MPS Input][ZIMPL Input]
* XpressMP [GAMS Input][MOSEL Input][MPS Input]

* Bonmin [AMPL Input]
* FilMINT [AMPL Input]
* MINLP [AMPL Input]
* SBB [GAMS Input]

Which benchmarks are available?

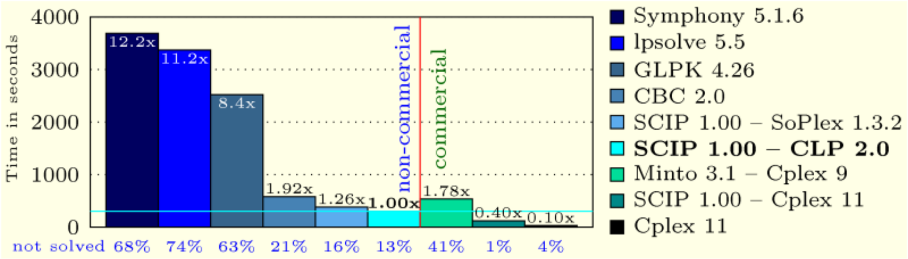
MILP Benchmark - free codes (2-27-2007)

Feasibility Benchmark - Feaspump vs CPLEX&SCIP (2-17-2008)

MI(QC)QP Benchmark (10-28-2007)

Graphical summary of our MILP benchmark

From The SCIP webpage `scip.zib.de`:



Geometric mean of results taken from the homepage of Hans Mittelmann (2/27/2008)

Thank you!

Questions?