



Weierstraß-Institut für Angewandte Analysis und Stochastik

MathFinance Workshop 2007, Frankfurt

Linear Monte Carlo upper bounds for Bermudan style derivatives

Pricing complex structured products



Leibniz
Gemeinschaft

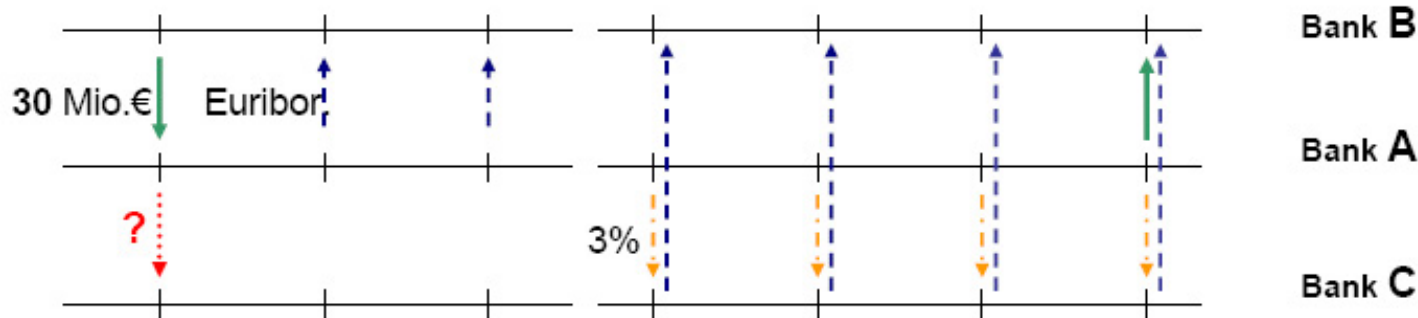
Bermudan callable products

Simple example: (Bermudan) callable interest rate swap

Euribor: Interest rate for a loan between banks

Contract I: A borrows from B 30 Mio. € over a period of 10 years and pays quarterly the 3M-Euribor.

Contract II: A buys from C a Bermudan swaption, i.e. the right to choose a payment date of contract I, from which on C pays quarterly the 3M-Euribor to B and receives a fixed payment of 3% from A.



'Exotic' example: cancelable snowball swap

Snowball swap: Instead of the floating spot rate the holder pays a starting coupon rate I over the first year and in the forthcoming years

$$(K + \text{previous coupon} - \text{spot rate})^+,$$

where the first coupon I and the strike rate K are specified in the contract

The investor is taking the view that interest rates will increase less aggressively than is being priced into the market

Cancelable snowball swap: The holder has the right to cancel this contract

What is the fair price of this cancelable product?



Bond Investment

Bester Anbieter strukturierter Produkte in der Schweiz
Euromoney Private Banking Awards 2006

3-jährige 2.75% Callable Snowball Note in CHF

15. Februar 2006

Indikative Konditionen*

Emittent	Credit Suisse, London Branch (A+/Aa3)
Lead Manager	Credit Suisse International
Zeichnungsfrist	bis am 16. März 2006, 12:00 MEZ
Anfangsvaluta	23. März 2006
Rückzahlung	23. März 2009 unterliegt einer Call-Klausel
Rückzahlung durch den Emittenten	100% Kapitalschutz
Coupon im 1. Semester	2.75% p.a.
Nachfolgende Coupons	siehe Coupontabelle rechts
LIBOR	6-Monats CHF LIBOR wie auf Telerats Seite 3740 publiziert, jeweils zwei Handelstage vor Ende der Couponperiode fixiert.
Coupondaten	halbjährlich (adjusted following)
Couponbasis	30/360, unadjusted
Stückelung	CHF 5'000
Ausgabepreis	100%
Verkaufskommission	0.50%
Sekundärmarkt	nur Ankaufslots (bid price) unter normalen Marktbedingungen Preis inklusive aufgelaufene Zinsen
Verkaufsrestriktionen	USA, US Personen, UK, EEA
Schweizer Valorennummer	2 428 261

Kurzbeschreibung

Callable Snowball Notes sind strukturierte Derivate. Sie bieten Ihnen als attraktive Obligationenalternative die Möglichkeit, Ihre Rendite zu verbessern. Diese Note ist für Sie interessant, wenn Sie den 6-Monats CHF LIBOR über die Laufzeit der Note nicht stark ansteigen sehen.

Funktionsweise

Die Höhe der Couponzahlungen hängt von der vorhergehenden Couponzahlung und vom 6-Monats CHF LIBOR ab, der jeweils 2 Handelstage vor Ende der halbjährlichen Couponperiode fixiert wird. Für das erste Semester wird jedoch ein fixer Coupon von 2.75% p.a. ausgeschüttet. Danach entspricht der Coupon dem vorhergehenden Coupon plus dem entsprechenden Step-up Zinssatz minus dem aktuellen 6-Monats CHF LIBOR. Der Step-up Zinssatz startet bei 1.25% und steigt danach bis auf 2.25% für die letzte Couponperiode im 3. Jahr an. Die untere Grenze der Couponzahlung liegt bei 0%, d.h. sie kann nicht negativ sein.

Couponschritte*

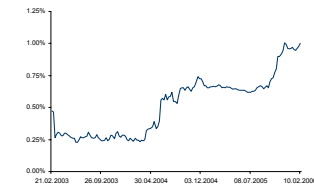
Semester	Coupon (p.a.)
1	2.75% fix
2	Vorhergehender Coupon + 1.25% - LIBOR
3	Vorhergehender Coupon + 1.50% - LIBOR
4	Vorhergehender Coupon + 1.75% - LIBOR
5	Vorhergehender Coupon + 2.00% - LIBOR
6	Vorhergehender Coupon + 2.25% - LIBOR

Quelle: Credit Suisse

Frühzeitige Rückzahlung

Die Note enthält eine Call-Klausel. Der Emittent hat das Recht, die Note erstmals am 23. September 2006 und danach an jedem weiteren Coupondatum vorzeitig zu pari (100%) zurückzahlen, mit einer Ankündigungsfrist von 5 Arbeitstagen an Euroclear.

6-Monats CHF LIBOR über die letzten 3 Jahre



Quelle: Bloomberg

* Wichtig:

Alle in diesem Dokument erwähnten Produktbedingungen sind nur indikativ und werden zur Anfangsfixierung bestätigt oder angepasst. Bitte beachten Sie, dass die Erläuterungen auf Seite 2 auch für diese Seite Gültigkeit haben.

Optimal stopping

▷ Mathematical problem:

Optimal stopping (calling) of a reward (cash-flow) process Z depending on an underlying (e.g. interest rate) process L

▷ Typical difficulties:

- L is usually **high dimensional**, for Libor interest rate models, $d = 10$ and up, so PDE methods do not work in general
- Z may only be virtually known, e.g. $Z_i = E^{\mathcal{F}_i} \sum_{j \geq i} C(L_j)$ for some pay-off function C , rather than simply $Z_i = C(L_i)$
- Z may be **path-dependent**

The standard Bermudan pricing problem

Consider an underlying process L in \mathbb{R}^D , e.g. a system of asset prices or Libor rates and a set of (future) dates $\mathbb{T} := \{T_0, T_1, \dots, T_k\}$

Bermudan derivative: An option to exercise a cashflow $C(T_\tau, L(T_\tau))$ at a future time $T_\tau \in \mathbb{T}$, to be decided by the option holder

Valuation: If N , with $N(0) = 1$, is some discounting numeraire and P the with N associated pricing measure, then with $Z_\tau := C(T_\tau, L(T_\tau))/N(T_\tau)$, the $t = 0$ price of the option is given by the **optimal stopping problem**

$$V_0 = \sup_{\tau \in \{0, \dots, k\}} E^{\mathcal{F}_0} Z_\tau,$$

where the supremum runs over all stopping indexes τ with respect to $\{\mathcal{F}_{T_i}, 0 \leq i \leq k\}$, where $(\mathcal{F}_t)_{t \geq 0}$ is the usual filtration generated by L .

Optimal stopping

At a future time point t , when the option is not exercised before t , the Bermudan option value is given by

$$V_t = N(t) \sup_{\tau \in \{\kappa(t), \dots, k\}} E^{\mathcal{F}_t} Z_\tau$$

with $\kappa(t) := \min\{m : T_m \geq t\}$.

The process

$$Y_t^* := \frac{V_t}{N(t)},$$

called the *Snell-envelope* process and is a supermartingale, i.e.

$$E^{\mathcal{F}_s} Y_t^* \leq Y_s^*$$

Canonical Solution by Backward Dynamic Programming

Set $Y_i^* := Y^*(T_i)$, $\mathcal{F}_i := \mathcal{F}_{T_i}$. At the last exercise date T_k we have

$$Y_k^* = Z_k$$

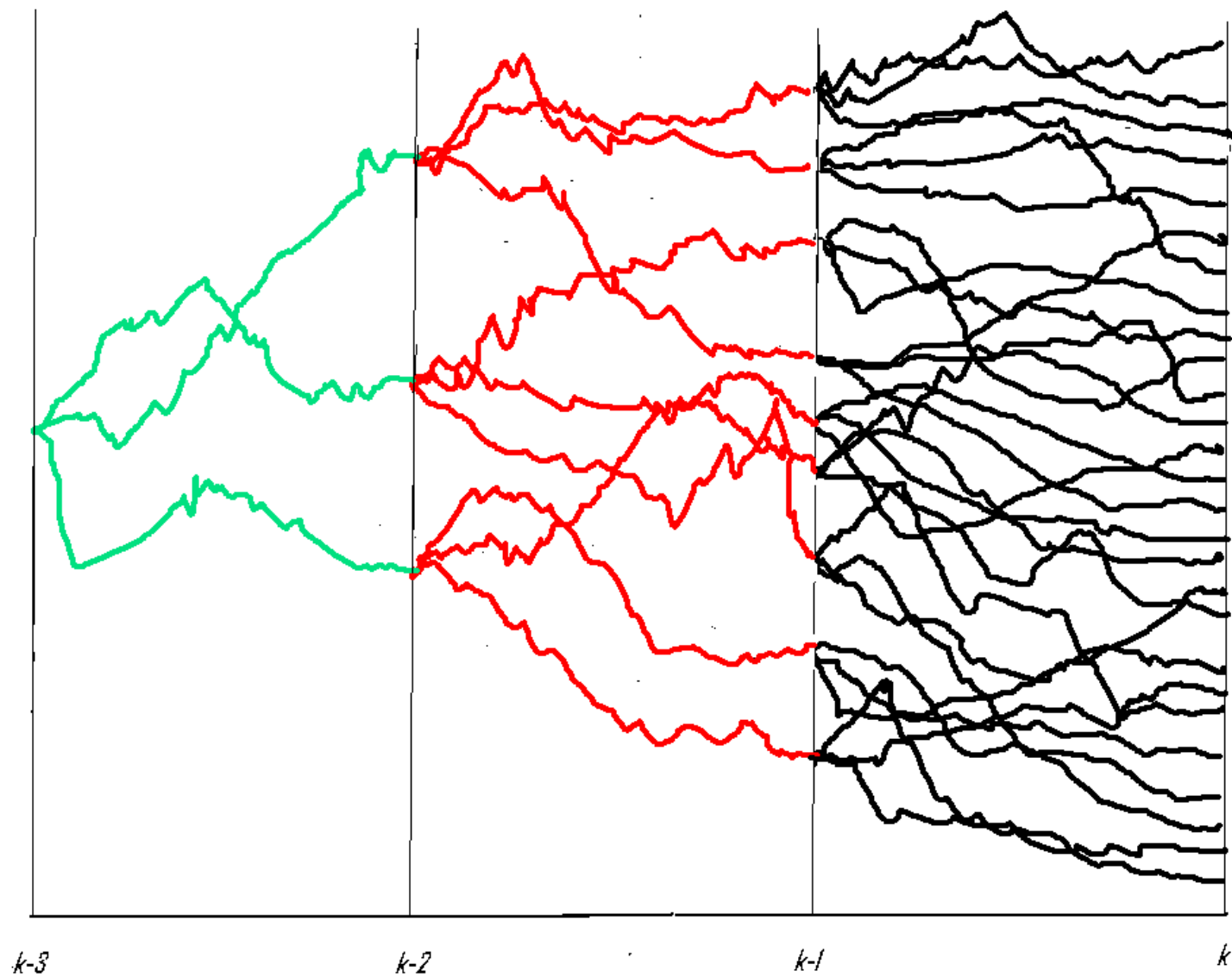
and for $0 \leq j < k$,

$$Y_j^* = \max \left(Z_j, E^{\mathcal{F}_j} Y_{j+1}^* \right).$$

→ Nested Monte Carlo simulation of the price Y_0^* would thus require N^k samples when conditional expectations are computed with N samples

Typically, $N=10000$, $k=10$ exercise opportunities, give 10^{40} samples !!

Optimal stopping



Construction of exercise policies

Let us consider a stopping family (policy) (τ_i) , which satisfies the **consistency conditions**:

$$i \leq \tau_i \leq k, \quad \tau_k = k, \quad \tau_i > i \Rightarrow \tau_i = \tau_{i+1}, \quad 0 \leq i < k,$$

and the corresponding **lower bound process** Y for the Snell envelope Y^* ,

$$Y_i := E^{\mathcal{F}_i} Z_{\tau_i} \leq Y_i^*$$

Example input policies:

▷ The policy

$$\tau_i := \inf\{j \geq i : L(T_j) \in G \subset \mathbb{R}^D\}$$

exercises when the underlying process L enters a certain region G .

Is the policy τ a good one ?

Upper approximations of the Snell envelope by Duality

The Dual Method

Consider a discrete martingale $(M_j)_{j=0,\dots,k}$ with $M_0 = 0$ with respect to the filtration $(\mathcal{F}_j)_{j=0,\dots,k}$. Following Rogers, Haugh and Kogan, we observe that

$$Y_0 := \sup_{\tau \in \{0,\dots,k\}} E^{\mathcal{F}_0} Z_\tau = \sup_{\tau \in \{0,\dots,k\}} E^{\mathcal{F}_0} [Z_\tau - M_\tau] \leq E^{\mathcal{F}_0} \max_{0 \leq j \leq k} [Z_j - M_j]$$

Hence the r.h.s. with an arbitrary martingale gives an upper bound for the Bermudan price Y_0 .

What martingale does lead to equality ?

Dual upper bounds

Theorem (Rogers (2001), Haugh & Kogan (2001))

Let M^* be the (unique) Doob-Meyer martingale part of $(Y_j^*)_{0 \leq j \leq k}$, i.e. M_j^* is an (\mathcal{F}_j) -martingale which satisfies

$$Y_j^* = Y_0^* + M_j^* - A_j^*, \quad j = 0, \dots, k,$$

with $M_0^* := A_0^* := 0$ and A^* being such that A_j^* is \mathcal{F}_{j-1} measurable for $j = 1, \dots, k$. Then we have

$$Y_0^* = E^{\mathcal{F}_0} \max_{0 \leq j \leq k} [Z_j - M_j^*].$$

Upper approximations of the Snell envelope using Riesz decomposition

Doob-Meyer decomposition

$$Y_j^* = Y_0^* + M_j^* - A_j^*, \quad j = 0, \dots, k,$$

and $Y_k^* = Z_k$ imply Riesz decomposition

$$Y_j^* = E^{\mathcal{F}_j} Z_k + E^{\mathcal{F}_j} (A_k^* - A_j^*)$$

Since $A_{i+1}^* - A_i^* = Y_i^* - E^{\mathcal{F}_i} Y_{i+1}^* = [Z_i - E^{\mathcal{F}_i} Y_{i+1}^*]^+$

$$Y_j^* = E^{\mathcal{F}_j} Z_k + E^{\mathcal{F}_j} \sum_{i=j}^{k-1} [Z_i - E^{\mathcal{F}_i} Y_{i+1}^*]^+$$

Riesz upper bounds

If Y_i is a lower approximation for Y_i^* then (Belomestny & Milstein (2005))

$$Y_j^{up} = E^{\mathcal{F}_j} Z_k + E^{\mathcal{F}_j} \sum_{i=j}^{k-1} [Z_i - E^{\mathcal{F}_i} Y_{i+1}]^+$$

is an upper approximation for Y_i^*

▷ **Monotonicity**

$$\tilde{Y}_i \geq Y_i \longrightarrow \tilde{Y}_i^{up} \leq Y_i^{up}$$

▷ **Locality**

Let $\{Y_i^\alpha, \alpha \in I_i\}$ be a family of local lower bounds at i , then

$$Y_j^{\alpha, up} = E^{\mathcal{F}_j} Z_k + E^{\mathcal{F}_j} \sum_{i=j}^{k-1} [Z_i - \max_{\alpha \in I_{i+1}} E^{\mathcal{F}_i} Y_{i+1}^\alpha]^+$$

is an upper bound

Fast upper bounds

For any martingale M_{T_j} , $0 \leq j \leq k$ with respect to the filtration $(\mathcal{F}_{T_j}; 0 \leq j \leq k)$ starting at $M_0 = 0$

$$Y_0^{up}(M) := E^{\mathcal{F}_0} \left[\max_{0 \leq j \leq k} (Z_{T_j} - M_{T_j}) \right]$$

is an upper bound for the price of the Bermudan option with discounted cash-flow Z_{T_j} .

Exact Bermudan price is attained at the martingale part M^* of the Snell envelope,

$$Y_{T_j}^* = Y_{T_0}^* + M_{T_j}^* - A_{T_j}^*,$$

$M_{T_0}^* = A_{T_0}^* = 0$ and $A_{T_j}^*$ is $\mathcal{F}_{T_{j-1}}$ measurable

Fast upper bounds

Assume that $Y_{T_j} = u(T_j, L(T_j))$ is an approximation of the Snell envelope $Y_{T_j}^*$, $0 \leq j \leq k$, with Doob decomposition

$$Y_{T_j} = Y_{T_0} + M_{T_j} - A_{T_j},$$

where $M_{T_0} = A_{T_0} = 0$ and A_{T_j} is $\mathcal{F}_{T_{j-1}}$ measurable.

It then holds:

$$Y_{T_{j+1}} - Y_{T_j} = M_{T_{j+1}} - M_{T_j} - A_{T_{j+1}} + A_{T_j}$$

$$M_{T_{j+1}} - M_{T_j} = Y_{T_{j+1}} - E^{T_j}[Y_{T_{j+1}}]$$

The computation of M_j by simulations leads to quadratic Monte Carlo for Y_0^{up}

Fast upper bounds

Assume the underlying process L to be Markovian, and the filtration \mathcal{F} to be generated by a D -dimensional Brownian motion W , then due to the [martingale representation theorem](#)

$$M_{T_j} =: \int_0^{T_j} H_t dW_t =: \int_0^{T_j} h(t, L(t)) dW_t, \quad j = 0, \dots, k,$$

where H_t is a square integrable and previsible process.

If we estimate function $h(\cdot, \cdot)$ and plug the estimate back we will get a martingale

Fast upper bounds

We are going to estimate H_t on partition $\pi = \{t_0, \dots, t_{\mathcal{I}}\}$ with $t_0 = 0$, $t_{\mathcal{I}} = T$, and $\{T_0, \dots, T_k\} \subset \pi$.

Write formally,

$$Y_{T_{j+1}} - Y_{T_j} \approx \sum_{t_l \in \pi; T_j \leq t_l < T_{j+1}} H_{t_l} \cdot (W_{t_{l+1}} - W_{t_l}) + A_{T_{j+1}} - A_{T_j}.$$

By multiplying both sides with $(W_{t_{i+1}}^d - W_{t_i}^d)$, $T_j \leq t_i < T_{j+1}$, and taking \mathcal{F}_{t_i} -conditional expectations, we get by the $\mathcal{F}_{T_{j+1}}$ -measurability of A_{T_j} ,

$$H_{t_i}^d \approx \frac{1}{t_{i+1} - t_i} E^{\mathcal{F}_{t_i}} \left[(W_{t_{i+1}}^d - W_{t_i}^d) \cdot Y_{T_{j+1}} \right],$$

and so

$$H_{t_i} \approx H_{t_i}^\pi := \frac{1}{\Delta_i^\pi} E^{\mathcal{F}_{t_i}} \left[(\Delta^\pi W_i)^\top Y_{T_{j+1}} \right], \quad T_j \leq t_i < T_{j+1},$$

with $\Delta_i^\pi := t_{i+1} - t_i$ and $\Delta^\pi W_i^d := W_{t_{i+1}}^d - W_{t_i}^d$.

Fast upper bounds

The corresponding approximation of the martingale M is

$$M_{T_j}^\pi := \sum_{t_i \in \pi; 0 \leq t_i < T_j} H_{t_i}^\pi \cdot \Delta^\pi W_i.$$

Theorem:

$$\lim_{|\pi| \rightarrow 0} E \left[\max_{0 \leq j \leq k} |M_{T_j}^\pi - M_{T_j}|^2 \right] = 0,$$

where $|\pi|$ denotes the mesh of π .

Fast upper bounds

The conditional expectations in the definition of H^π are functions of $L(t_i)$:

$$h^\pi(t_i, L(t_i)) = \frac{1}{\Delta_i^\pi} E^{L(t_i)} [(\Delta^\pi W_i)^\top u(T_{j+1}, L(T_{j+1}))], \quad T_j \leq t_i < T_{j+1}$$

and can be computed by regression.

Take basis functions

$$\psi(t_i, \cdot) = (\psi_r(t_i, \cdot), r = 1, \dots, R)$$

and N independent samples

$$(t_i, {}_n L(t_i)), \quad n = 1, \dots, N$$

from $L(t_i)$ constructed using the Brownian increments $\Delta_n^\pi W_i$, $n = 1, \dots, N$.

Fast upper bounds

Construct the regression matrix $A_{t_i}^\oplus := (A_{t_i}^\top A_{t_i})^{-1} A_{t_i}^\top$, where

$$A_{t_i} = (\psi_r(t_i, {}_n L(t_i)))_{n=1, \dots, N, r=1, \dots, R}.$$

Then define

$$\widehat{h}^\pi(t_i, x) = \psi(t_i, x) A_{t_i}^\oplus \left(\frac{\Delta_i^\pi W_i}{\Delta_i^\pi} \cdot Y_{T_{j+1}} \right) =: \psi(t_i, x) \widehat{\beta}_{t_i},$$

where

$$\left(\frac{\Delta_i^\pi W_i}{\Delta_i^\pi} \cdot Y_{T_{j+1}} \right) = \left(\frac{\Delta_n^\pi W_i^d}{\Delta_i^\pi} {}_n Y_{T_{j+1}} \right)_{n=1, \dots, N, d=1, \dots, D},$$

${}_n Y_{T_{j+1}} := u(T_{j+1}, {}_n L(T_{j+1}))$, and $\widehat{\beta}_{t_i}$ is the $R \times D$ matrix of estimated regression coefficients at time t_i .

True linear Monte Carlo upper bound

$$\widehat{Y}^{up}(\widehat{M}^\pi) = \frac{1}{\widetilde{N}} \sum_{n=1}^{\widetilde{N}} \max_{0 \leq j \leq k} \left[z(T_j, {}_n\widetilde{L}(T_j)) - \sum_{t_i \in \pi; 0 \leq t_i < T_j} \widehat{h}^\pi(t_i, {}_n\widetilde{L}(T_j)) \cdot (\Delta_n^\pi \widetilde{W}_i) \right],$$

by simulating new paths

$$({}_n\widetilde{L}(T_j), \Delta_n^\pi \widetilde{W}_i), \quad 0 \leq t_i < T_j, \quad n = 1, \dots, \widetilde{N}.$$

$$\widetilde{M}_j = \sum_{t_i \in \pi; 0 \leq t_i < T_j} \widehat{h}^\pi(t_i, \widetilde{L}(T_j)) \cdot (\Delta^\pi \widetilde{W}_i)$$

is always a martingale, so the upper bound is true!

Application: Max Call on D assets

Black-Scholes model:

$$dX_t^d = (r - \delta)X_t^d dt + \sigma X_t^d dW_t^d, \quad d = 1, \dots, D,$$

Pay-off:

$$Z_t := z(X_t) := (\max(X_t^1, \dots, X_t^D) - \kappa)^+.$$

$T_k = 3\text{yr}$, $k = 9$ (ex. dates), $\kappa = 100$, $r = 0.05$, $\sigma = 0.2$, $\delta = 0.1$, $D = 2$ and different x_0

x_0	Lower Bound Y_0	Upper Bound $Y_0^{up}(\widehat{M}^\pi)$	Upper Bound $Y_{10^4, 200}^{up}(0)$	Upper Bound $Y_{10^4, 100}^{up}(0)$
90	7.9751 ± 0.139	8.4963 ± 0.052	8.231	8.501 ± 0.06
100	13.883 ± 0.177	14.315 ± 0.073	14.18	14.35 ± 0.07
110	21.291 ± 0.205	21.772 ± 0.095	21.68	21.79 ± 0.11

Application: Bermudan swaptions

A first numerical example: Bermudan swaptions in the LIBOR market model

Consider the Libor Market Model with respect to a tenor structure $0 < T_1 < T_2 < \dots < T_n$, e.g. in the spot Libor measure P^* induced by the numeraire

$$B^*(t) := \frac{B_{m(t)}(t)}{B_1(0)} \prod_{i=0}^{m(t)-1} (1 + \delta_i L_i(T_i))$$

with $m(t) := \min\{m : T_m \geq t\}$.

The dynamics of the forward Libor $L_i(t)$ is given by a system of SDE's

$$dL_i = \sum_{j=m(t)}^i \frac{\delta_j L_i L_j \gamma_i \cdot \gamma_j}{1 + \delta_j L_j} dt + L_i \gamma_i \cdot dW^*.$$

Here $\delta_i = T_{i+1} - T_i$ are day count fractions, and

$$t \rightarrow \gamma_i(t) = (\gamma_{i,1}(t), \dots, \gamma_{i,d}(t))$$

are deterministic volatility vector functions defined in $[0, T_i]$, called factor loadings.

Application: Bermudan swaptions

A (payer) Swaption over a period $[T_i, T_n]$, $1 \leq i \leq k$. A swaption contract with maturity T_i and strike θ with principal \$1 gives the right to contract at T_i for paying a fixed coupon θ and receiving floating Libor at the settlement dates T_{i+1}, \dots, T_n . So by this definition, its cashflow at maturity is

$$S_{i,n}(T_i) := \left(\sum_{j=i}^{n-1} B_{j+1}(T_i) \delta_j (L_j(T_i) - \theta) \right)^+ .$$

A Bermudan Swaption gives the the right to exercise a cashflow

$$C_{T_\tau} := S_{\tau,n}(T_\tau)$$

at an exercise date $T_\tau \in \{T_1, \dots, T_n\}$ to be decided by the option holder.

Application: Bermudan swaptions

Prices of bermudan swaptions $\times 10^4$

D	θ	Y_0	$Y_0^{up}(\widehat{M}^\pi)$
40	0.08	1094.8 \pm 1.2	1096.1 \pm 2.0
	0.10	338.2 \pm 1.0	341.2 \pm 1.3
	0.12	96.4 \pm 0.5	100.0 \pm 0.6
10	0.08	1096.3 \pm 1.3	1096.6 \pm 2.0
	0.10	344.3 \pm 1.0	346.7 \pm 1.3
	0.12	101.7 \pm 0.6	104.9 \pm 0.7
1	0.08	1108.1 \pm 1.5	1110.5 \pm 2.4
	0.10	381.7 \pm 1.2	384.7 \pm 1.6
	0.12	121.2 \pm 0.7	123.1 \pm 0.8

Path dependent callable products

Consider a **path dependent cancelable contract** which generates (possibly negative) cash-flows C_1, \dots, C_τ up to a cancellation date τ . The cash-flows of this contract are equivalent to an aggregated cash-flow at cancellation date,

$$B_*(\mathcal{T}_\tau) \mathcal{Z}_\tau := B_*(\mathcal{T}_\tau) \sum_{j=1}^{\tau} Z_j,$$

with $Z_i := C_i / B_*(\mathcal{T}_i)$ being discounted cash-flows w.r.t. the numeraire B_* .

Product price at time zero:

$$V_0^{cancel} := \sup_{\tau \in \{1, \dots, k\}} E^{\mathcal{F}_0} \mathcal{Z}_\tau = \sup_{\tau \in \{1, \dots, k\}} E^{\mathcal{F}_0} \sum_{j=1}^{\tau} Z_j,$$

where the sup is taken over all stopping indices with values in the $\{1, \dots, k\}$.

Path-dependent callables

A path dependent **callable** contract generates

$$C_{\tau+1}, \dots, C_k$$

when called at τ . It is equivalent to the sum of a non-callable and a cancelable one (and vice versa):

$$\begin{aligned} V_0^{call} &:= \sup_{\tau \in \{1, \dots, k\}} E^{\mathcal{F}_0} \sum_{j=\tau+1}^k Z_j \\ &= E^{\mathcal{F}_0} \sum_{j=1}^k Z_j + \sup_{\tau \in \{1, \dots, k\}} E^{\mathcal{F}_0} \sum_{j=1}^{\tau} (-Z_j) \end{aligned}$$

Example: The cancelable snowball swap

Pays semi-annually a constant rate I over the first year and in the forthcoming years $(\text{Previous coupon} + A - \text{Libor})^+$, semi-annually, where A is given in the contract. For this case we take

$$\begin{aligned}K_i &:= I, \quad i = 0, 1, \\K_i &:= (K_{i-1} + A_i - L_i(T_i))^+, \quad i = 2, \dots, n - 1,\end{aligned}$$

with $A_2 := 0.03$, $A_{i+1} = A_i$ for even i , $A_{i+1} = A_i + s$ for odd i . Cancellation is allowed for $2 \leq \tau < n$, $n = 20$ (10 years)

Effective discounted cashflows at T_j :

$$Z_j := \frac{(L_{j-1}(T_{j-1}) - K_{j-1}) \delta_{j-1}}{B_*(T_j)},$$

hence aggregated up to cancellation $Z_\tau = \sum_{j=1}^{\tau} Z_j$.

Application: Snowballs

Tenor structure, initial Libor curve and factor loadings

Tenors	0.0	0.5	1	1.5	2	2.5
L_0	0.023	0.025	0.027	0.027	0.031	0.031
c_i		0.153	0.143	0.14	0.140	0.139
Tenors	3	3.5	4	4.5	5	5.5
L_0	0.033	0.034	0.036	0.036	0.038	0.039
c_i	0.138	0.137	0.136	0.135	0.134	0.132

Application: Snowballs

Prices of cancellable snowballs $\times 10^4$ in dependence on s

s	Y_0	$Y_0^{up}(\widehat{M}^\pi)$
0.005	64.8 ± 2.4	67.4 ± 2.2
0.004	101.9 ± 2.3	107.3 ± 1.9
0.003	139.8 ± 2.2	143.3 ± 1.7