Credit Risk under Noisy Asset Information

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A Model with Noisy Asset Information

- Consider a structural model
- Fact: firm value $(V_t)_t$ is not observable !
- Investors use other sources of information:
  - Accounting information (Duffie & Lando 2001)
  - Information collected from newspapers, etc

$\Rightarrow$ Incomplete information leads to a filter problem

Goal of this work:

1. Fundamental relationship of equity and debt under incomplete information (joint with R. Frey)
2. Tractable framework which incorporates sudden information updates using shot noise models

Information Structure

What kind of information is available to the investor ?

1. Information on the default state
2. News on the company at random times
   - If the company is in trouble $\Rightarrow$ more frequent news
     ie news intensity depends on healthiness of company
   - News is discrete, for example good / bad or rating
3. Dividends depend on the firm value

Fundamental value of equity under full information

- Denote firm value by $(V_t)_{t\geq 0}$, default by $\tau := \inf\{t > 0 : V_t \leq K\}$
- Dividends D_n, paid at T_n, $n = 1, 2, \dots$, are noisy signals of V:

$$D_n = \delta_n V_{T_n-}$$

  δ_n i.i.d. $\sim \nu_\delta(dx)$

- Full information: $\mathcal{F}_t = \sigma(V_s : s \leq t, (T_n, D_n) : T_n \leq t)$
- Set $R_t = \sum_{T_n \leq t} \delta_n \cdot \kappa$, $\kappa \in [0, 1]$. Under some EMM Q

$$dV_t = V_{t-}\mu\, dt + V_{t-}\sigma\, dW_t - V_{t-} dR_t$$

- Equity is discounted value of the paid dividends,

$$S_t = 1_{\{\tau > t\}}\, \mathbb{E}^Q\Big(\int_t^\tau e^{-r(s-t)} V_{s-} dR_s \Big| \mathcal{F}_t \Big). \tag{1}$$

Make it easy:

- Assume T_n are jump times from a Poisson process with intensity λ
- Denote $\bar{\delta} := \mathbb{E}^Q(\delta_1)$

Proposition

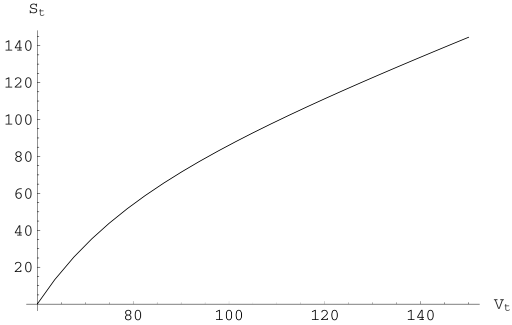
Under $\mu < \lambda\bar{\delta} + r$, $\kappa = 1$

$$S_t = \frac{\lambda\bar{\delta}}{r + \lambda\bar{\delta} - \mu}\Big[\frac{V_t}{K} - \Big(\frac{V_t}{K}\Big)^{\alpha^*} K\Big], \tag{2}$$

where $\alpha^* < 0$ *such that*

$$0 = \alpha\mu - r + \frac{1}{2}\sigma^2\alpha(\alpha - 1) + \lambda\Big[\int_0^1 (1-x)^\alpha \nu_\delta(dx) - 1\Big]$$

- In the simulation we use a Beta-distributed δ.



Note that S is a concave function of V

Pricing Debt

- On $\{\tau > T\}$ derive

$$\mathbb{P}(\tau > T | V_t) = \mathbb{P}\Big(\inf_{s \in (t,T]} V_s > K | V_t \Big)$$

- Default probability leads to prices for Bonds and CDS

- Summarizing, we can price equity and debt under full information

The case of incomplete information

- News are modeled as $(I_n)_{n=1,2,\ldots}$, I_n occurring at $\tilde{T}_n$,
- Specify $\mathbb{P}(I_n = i | V_{T_n} = v) = \nu_I(i, v)$ for $i \in \{i_1, \ldots, i_M\}$
- Information available to the investor is

$$\mathcal{H}_t := \left\{ 1_{\{V_s > K\}} : s \le t \,;\, (T_n, D_n) : T_n \le t \,;\, (\tilde{T}_m, I_m) : \tilde{T}_m \le t \right\}$$

- Value of equity is

$$S_t = 1_{\{\tau > t\}} \, \mathbb{E}^Q \Big[\underbrace{\mathbb{E}^Q \Big(\int_t^\tau e^{-r(s-t)} V_{s-} dR_s \Big| \mathcal{F}_t \Big)}_{(*)} \Big| \mathcal{H}_t \Big].$$

- $(*) = f(V_t)$ already computed (full information case)

$\Rightarrow$ Need: conditional distribution of V_t given $\mathcal{H}_t$

- Analogously for default probability, bond and CDS

Find an approximate solution for the filter problem

- Consider a discrete time approximation, $t_k = \Delta k$, for fixed Δ
- Use discrete-time, discrete space Markov chain (V_k)
- State space $M^0 = \{m_1^0, \dots, m_N^0\}$, transition probabilities p_{ij}^0
- Discrete Information:

$$\mathcal{H}_k^\Delta := \sigma\Big(V_i > K(t_i) : i \leq k\,,\, (T_n, D_n) : T_n \leq t_k\,,\, (\tilde{T}_m, I_m) : \tilde{T}_m \leq t_k\Big).$$

Conditional Distribution of (V_k)

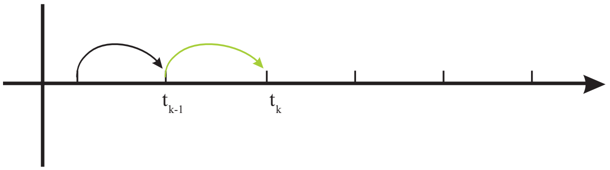
Information at time t_k after d dividends is

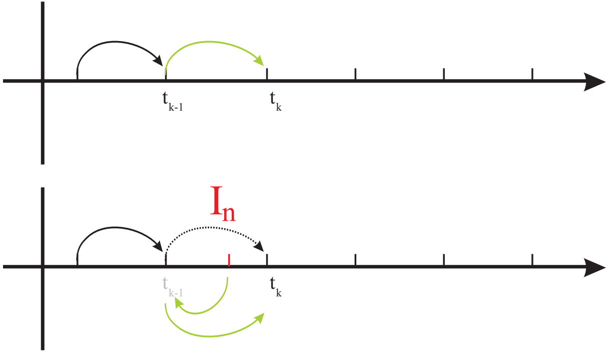
$$A_k := \{V_i > K(t_i) : i \le k\}, \quad \mathcal{I}_k := \sigma\big((T_n, D_n) : T_n \le t_k\,,\, (\tilde{T}_m, I_m) : \tilde{T}_m \le t_k\big)$$

Then, on $\{T_d \le t_k, T_{d+1} > t_k\}$,

$$\begin{aligned} \mathbb{P}\big(V_k = m_j^0 \big| \mathcal{H}_k^\Delta\big) &= \mathbb{P}\big(V_k = m_j^0 \big| 1_{A_k}, \mathcal{D}_k\big) \\ &= \frac{\mathbb{P}\big(V_k = m_j^0, A_k \big| \mathcal{D}_k\big)}{\mathbb{P}\big(A_k \big| \mathcal{D}_k\big)} =: \frac{\pi_j^k}{\sum_{i=1}^N \pi_i^k}. \end{aligned}$$

Thus, we need only the unnormalized probabilities π_j^k


t_{k-1}
t_k



t_{k-1}
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t_{k-1}
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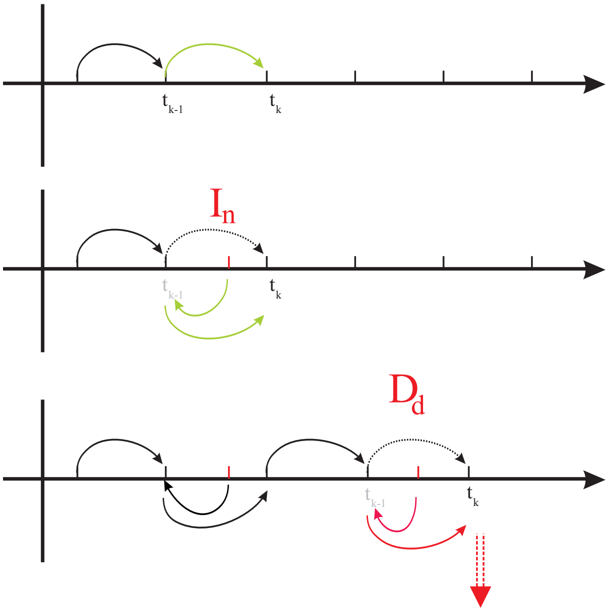


t k-1
t k
I n
t k-1
t k
D d
t k-1
t k

Proposition

If Information Intensity does not depend on the firm value, we have:

1. *If t_k is not a dividend date and no information arrives, then*

$$\pi_j^k = 1_{\{m_j^k > K\}} \sum_{i=1}^{N} p_{ij}^k \, \pi_i^{k-1}. \tag{3}$$

2. *If t_k is not a dividend date but $\{t_{k-1} < \tilde{T}_i \leq t_k\}$, then*

$$\pi_j^k = 1_{\{m_j^k > K\}} \, \tilde{\pi}_j^k \, \nu_I(I_i, m_j^k),$$

   where $\tilde{\pi}_j^k$ are computed according to (3)

3. *On $\{t_{k-1} < T_d \leq t_k\}$, but no information we have*

$$\pi_j^k = 1_{\{m_j^{k-1} > K + D_d\}} \, \tilde{\pi}_j^k \, \frac{1}{m_j^{k-1}} \nu_\delta\Big(\frac{D_d}{m_j^{k-1}}\Big),$$

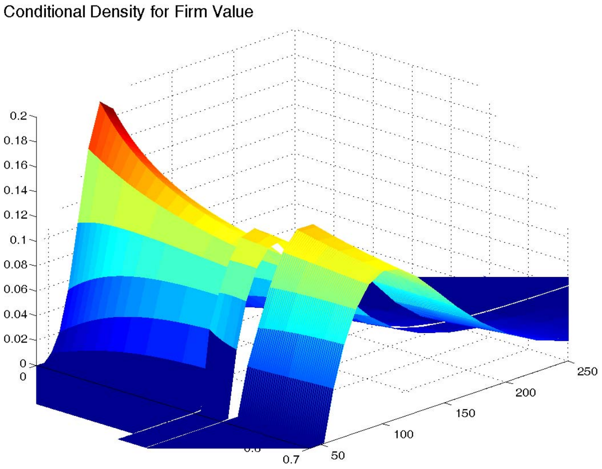
   where $\tilde{\pi}_j^k$ are computed according to (3)

4. *Combination of information and dividends ...*

Incorporating additional information is also possible (see paper):

- News intensity depends on the firm value
- Correlated index is equivalent to time dependent boundary

Analysis of the filter in practice



We have the following result:

Lemma

If the stock price is a concave function of the firm value (under full information) then neglecting incomplete information leads to a underestimation of the firm value.

Proof.

We have from Proposition 1, that

$$S_t = f(V_t),$$

where f is a concave function. Using the stock price as statistic for the firm value and neglecting incomplete information leads to

$$\hat{V}_t = f^{-1}(S_t) = f^{-1}\big(\mathbb{E}(f(V_t)|\mathcal{H}_t)\big).$$

f is monotonically increasing, so $\hat{V}_t \leq V_t$ is equivalent to

$$f(\hat{V}_t) = \mathbb{E}(f(V_t)|\mathcal{H}_t) \leq f\big(\mathbb{E}(V_t|\mathcal{H}_t)\big),$$

which follows using Jensen's inequality. ∎

A Shot Noise Model for Pricing Credit

Motivated by the previous results, assume the intensity follows

$$\lambda_t = \tilde{\lambda}_t + J_t,$$

where $\tilde{\lambda}$ is some diffusion component (eg general quadratic) and

$$J_t = \sum_{i=1}^{\tilde{N}_t} Y_i \cdot h(t - \tilde{\tau}_i),$$

$\tilde{N}$ is Poisson process with jumps $\tilde{\tau}_i$.
Typical examples:

$$h(t) = e^{-at}$$
$$h(t) = 1_{[0,a]}$$

or more general forms. Markovianity only for the 1st case!

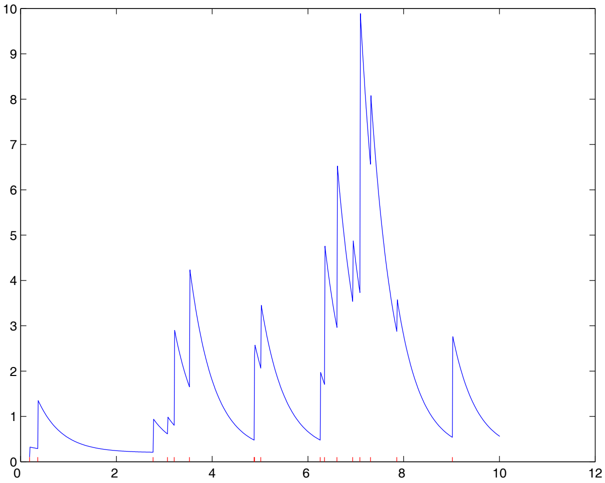


Figure: Possible realization of the process J with $h(x) = e^{-0.5x}$ and χ^2_2-distributed Y_i.

Credit Portfolios with Clustering

- In Gaspar, S (2006) we apply this approach to credit PFs and CDOs.
- Consider $k = 1, \dots, n$ credit risky with individual intensities

$$\lambda_t^k = \mu_t^k + \epsilon^k \mu_t^c.$$

  Here μ^k and μ^c have the shot noise feature and the diffusions are general quadratic.

⇒ Obtain Markovian model with closed-form solutions for lots of credit derivatives, etc.

- Generalizes Duffie, Garleanu (2001) and allows for default clustering, ie high default correlation.

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  Econometrica 2001
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