

# On the Cost of Delayed Currency Fixing Announcements

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## **Abstract**

In Foreign Exchange Markets vanilla and barrier options are traded frequently. The market standard is a cutoff time of 10:00 a.m. in New York for the strike of vanillas and a knock-out event based on a continuously observed barrier in the inter bank market. However, many clients, particularly from Italy, prefer the cutoff and knock-out event to be based on the fixing published by the European Central Bank on the Reuters Page ECB37. These barrier options are called discretely monitored barrier options. While these options can be priced in several models by various techniques, the ECB source of the fixing causes two problems. First of all, it is not tradable, and secondly it is published with a delay of about 10 - 20 minutes. We examine here the effect of these problems on the hedge of those options and consequently suggest a cost based on the additional uncertainty encountered.

## 1 Introduction

### 1.1 The Currency Fixing of the European Central Bank

The European Central Bank (ECB) sets currency fixings every working day in Frankfurt at 2:15 p.m. Frankfurt time. The actual procedure of this fixing is done by observing the spot rates in the inter bank market, in which the ECB also participates. Traders of the ECB in various locations get together to decide on how to set the fixing. The quantity quoted is not a bid price or an offer price the ECB or anybody else necessarily trades at, but is rather used for statistical and official means, for instance tax computation or economic research.

Corporate treasurers often prefer an independent source for currency exchange rates that provides a reference rate for their underlying under consideration. This way they are not bound to their own bank that might move the quoted cut-off rate in favor of their own position. The key features to stress are the following.

1. The ECB fixing is *not* tradable.
2. The ECB fixing is published with a delay of 10-20 minutes.

In this paper we analyze the impact on the value for the short position, as the problems mentioned above impose additional uncertainty when it comes to determining a proper hedge. The currency-pairs under consideration are EUR-USD, USD-JPY, USD-CHF and GBP-USD. The instruments of interest are vanillas, European style knock out barrier options and discretely monitored barrier options. Most of the hedging error is expected in the case of jumps in the payoff of the option, which is why we restrict ourselves to the liquidly traded up-and-out-Call-Option.

### 1.2 Model and Payoff

To model the exchange rate we choose a geometric Brownian motion,

$$dS_t = S_t[(r_d - r_f) dt + \sigma dW_t], \quad (1)$$

under the risk-neutral measure. As usual,  $r_d$  denotes the domestic interest rate,  $r_f$  the foreign interest rate,  $\sigma$  the volatility. These parameters are assumed to be constant in this paper. For contract parameters maturity in years  $T$ , strike  $K$  and knock-out barrier  $B$ , fixing schedule  $0 = t_0 < t_1 < t_2 \dots, t_n = T$ , the payoffs for the vanilla and for a discretely monitored up-and-out barrier option under consideration are

$$V(F_T, T) = (\phi(F_T - K))^+, \quad (2)$$

$$V(F, T) = (\phi(F_T - K))^+ \mathbb{I}_{\{\max(F_{t_0}, \dots, F_{t_n}) < B\}}, \quad (3)$$

where  $F_t$  denotes the fixing of the underlying exchange rate at time  $t$ ,  $\mathbb{I}$  the indicator function and  $\phi$  a put-call indicator taking the value +1 for a call and -1 for a put. Of course,  $F_t$  is usually close to  $S_t$ , the spot at time  $t$ , but it may differ as well. We start with payoffs

$$V(S_T, T) = (\phi(S_T - K))^+, \quad (4)$$

$$V(S, T) = (\phi(S_T - K))^+ \mathbb{I}_{\{\max(S_{t_0}, \dots, S_{t_n}) < B\}}, \quad (5)$$

whose values are explicitly known in the Black-Scholes model.

### 1.3 Analysis Procedure

1. We simulate the spot process with a Monte Carlo simulation using an Euler-discretisation. Furthermore, we use a *Mersenne Twister* pseudo random number generator by Takuji Nishimura and Makoto Matsumoto [3] and a library to compute the inverse of the normal cumulative distribution function written by Barry W. Brown et al. [1].
2. We model the ECB-fixing  $F_t$  by

$$F_t = S_t + \varphi, \quad \varphi \in \mathcal{N}(\mu, \sigma), \quad (6)$$

where  $\mu$  and  $\sigma$  are estimated from historic data. Note that  $F_t$  denotes the ECB-fixing at time  $t$ , which is nonetheless only announced 10 - 20 minutes later. We denote this time delay by  $\Delta T$ . This means that we model the error, i.e. the difference of fixing and traded spot, as a normally distributed random variable. The estimated values for the mean and the standard-deviation of the quantity Spot - ECB Fixing from historic time series are listed in Table 1.

Currency pair	Expected value	Standard deviation	Time horizon
EUR / USD	-3.125E-6	0.0001264	23.6 - 08.8.04
USD / YEN	-4.883E-3	0.0134583	22.6 - 26.8.04
USD / CHF	-1.424E-5	0.0001677	11.5 - 26.8.04
GBP / USD	-6.262E-6	0.0002596	04.5 - 26.8.04

Table 1: Estimated values for mean and standard-deviations of the quantity Spot - ECB-fixing from historic time series

3. We evaluate the payoffs for barrier options for each path and run the simulations with the appropriate delta hedge quantities to hold. Then we compute for each path the error encountered due the fixing being different from the spot, and then average over all paths.
4. We do this for various currency pairs, parameter scenarios, varying the rates, volatilities, maturities, barriers and strikes. We expect a significant impact particularly for reverse knock-out barrier options due to the jump of the payoff and hence the large delta hedge quantity.

## 2 Error Estimation

Note that since we expect the resulting errors to be fairly small, we introduce a bid/offer-spread  $\delta$  for the spot, which is of the size of 2 pips in the inter bank market. We consider the following options:

## 2.1 European up-and-out Call

To determine the possible hedging error we propose the following to be appropriate. Note that the error is measured for a nominal of 1 unit of the underlying. We consider three cases.

1. Let  $S_T \leq K$ . In this case, the seller who is short the option decides not to hedge as the option is probably out of the money, i.e.  $\Delta = 0$ . If the option turns out to be in the money, i.e.  $F_T > K$ , the holder of the short position faces a P&L of  $K - (S(T + \Delta T) + \delta)$  (units of the base currency).
2. Let  $S_T > K$  and  $S_T < B$ . Hence, one assumes that the option is in the money and delta is 1. If now  $F_T \leq K$  or  $F_T \geq B$ , there is a P&L of  $S(T + \Delta T) - (S(T) + \delta)$ .
3. Let  $S_T \geq B$  and  $F_T < B$ . Here we have a P&L of  $K - (S(T + \Delta T) + \delta)$ . Note that other than in the first case, this P&L is of order  $K - B$  due to the jump in the payoff.

## 2.2 Discretely monitored up-and-out Call

We consider a time to maturity of one year, i.e. 250 knock-out-events, that is to say, the possible knock out occurs every working day at 2:15 p.m. Frankfurt time, when the ECB fixes the reference rate of the underlying currency pair. We propose the following error determination to be appropriate. First of all, we adopt the procedure above for the maturity time. In addition, we consider every knock-out-event and examine the following cases.

1. Let  $S_t < B$  and  $F_t \geq B$ . Here we unwind our hedge with delay and encounter a P&L of

$$\Delta(S_t) \cdot (S_{t+\Delta T} - S_t), \quad (7)$$

where  $\Delta(S_t)$  denotes the theoretical delta of the option under consideration, if the spot is at  $S_t$ . To see this, it is important to note, that the theoretical delta is negative if the underlying is near the barrier  $B$ . In this way, the seller of the option has been short the underlying at time  $t$  and must buy it in  $t + \Delta T$  minutes to close out the hedge. Therefore, he makes profit if the underlying is cheaper in  $t + \Delta T$ , which is reflected in our formula. We shall elaborate later how to compute the theoretical delta, but we would like to point out that whenever we need a spot price at time  $t$  to calculate such a delta or to compute the value of a hedge, we refer to  $S$  as the tradable instrument instead of the contractually specified underlying  $F$  in order to account for the ECB fixing being non-tradable.

2. Let  $S_t \geq B$  and  $F_t < B$ . Here the seller of the option closed out the hedge at time  $t$ , though she shouldn't have done so, and in  $t + \Delta T$  she needs to build a new hedge. Note again that the theoretical delta is negative. This means that at time  $t$  the seller bought the underlying with the according theoretical delta-quantity, and in  $t + \Delta T$  she goes short the underlying

with the appropriate new delta-quantity. The profit and loss (P&L) is calculated via

$$\text{P\&L} = \Delta(S_t) \cdot (S_t + \delta) - \Delta(S_{t+\Delta T}) \cdot S_{t+\Delta T} \quad (8)$$

The other cases seem to be irrelevant as it is always fine to hedge with the theoretical hedge quantity as long as the underlying is far away from the knock-out-barrier.

### 2.3 Calculating the Delta-Hedge Quantity

In order to compute the theoretical delta for the discretely monitored up-and-out call, for which no closed-form solution is known, in acceptable time and precision, we refer to an approximation proposed by Per Hörfelt in [2], which works in the following way. Assume the value of the spot is observed at times  $iT/n$ ,  $i = 0, \dots, n$ , and the payoff of the discretely monitored up-and-out call is given by Equation (5). We define the value and abbreviations

$$\theta_{\pm} \triangleq \frac{r_d - r_f \pm \sigma^2/2}{\sigma} \sqrt{T}, \quad (9)$$

$$c \triangleq \frac{\ln(K/S_0)}{\sigma \sqrt{T}}, \quad (10)$$

$$d \triangleq \frac{\ln(B/S_0)}{\sigma \sqrt{T}}, \quad (11)$$

$$\beta \triangleq -\zeta(1/2)/\sqrt{(2\pi)} \approx 0.5826, \quad (12)$$

where  $\zeta$  denotes the Riemann zeta function. We define the function

$$F_+(a, b; \theta) \triangleq \mathcal{N}(a - \theta) - e^{2b\theta} \mathcal{N}(a - 2b - \theta) \quad (13)$$

and obtain for the value of the discretely monitored up-and-out call

$$\begin{aligned} V(S_0, 0) \approx & S_0 e^{-r_f T} [F_+(d, d + \beta/\sqrt{n}; \theta_+) - F_+(c, d + \beta/\sqrt{n}; \theta_+)] \\ & - K e^{-r_d T} [F_+(d, d + \beta/\sqrt{n}; \theta_-) - F_+(c, d + \beta/\sqrt{n}; \theta_-)]. \end{aligned} \quad (14)$$

Using this approximation for the value, we take a finite difference approach for the computation of the theoretical delta

$$\Delta = V_S(S, t) \approx \frac{V(S + \epsilon, t) - V(S - \epsilon, t)}{2\epsilon}. \quad (15)$$

## 3 Analysis of EUR-USD

Considering the simulations for a maturity  $T$  of one year, huge hedging errors can obviously only occur near the barrier. The influence of the strike is comparatively small, as we discussed the error determination procedure above. In this way we chose the values listed in Table 2 to remain constant and only to vary the barrier.

Spot	1.2100
Strike	1.1800
Trading days	250
domestic interest rate	2.17% (USD)
Foreign interest rate	2.27% (EUR)
Time to maturity	1 year
Notional	1,000,000 EUR

Table 2: EUR-USD Testing parameters

The data produced by Monte Carlo simulations of the profit and loss turned out to be quite unstable, but a simple regression does reveal a certain trend in the relationships under consideration. Figure 1 shows how additional hedge costs for the short position develop. Noticeably the additional costs for the short position are negligible.

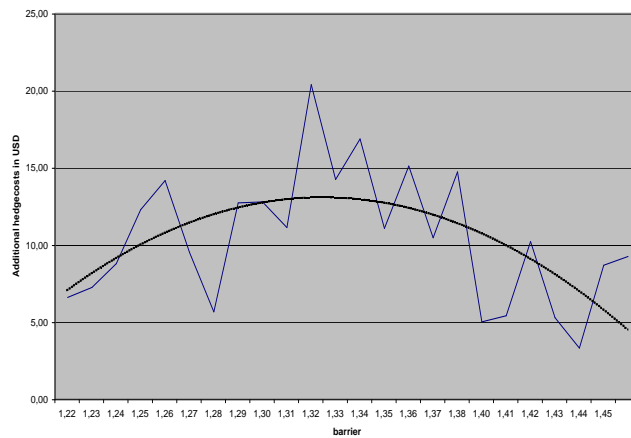


Figure 1: Additional hedge costs for the short position of a discretely monitored up-and-out Call in EUR-USD

Figure 2 shows how the probability of a miss-hedge depends on the position of the barrier. The blue line represents real data, the black line is an exponential approximation whose very good fit affirms how fast the probability falls down to zero.

In Figure 3 we plot the barrier against the ratio Hedging Error / TV of the up-and-out Call and the ECB-fixing as underlying. This relationship is an

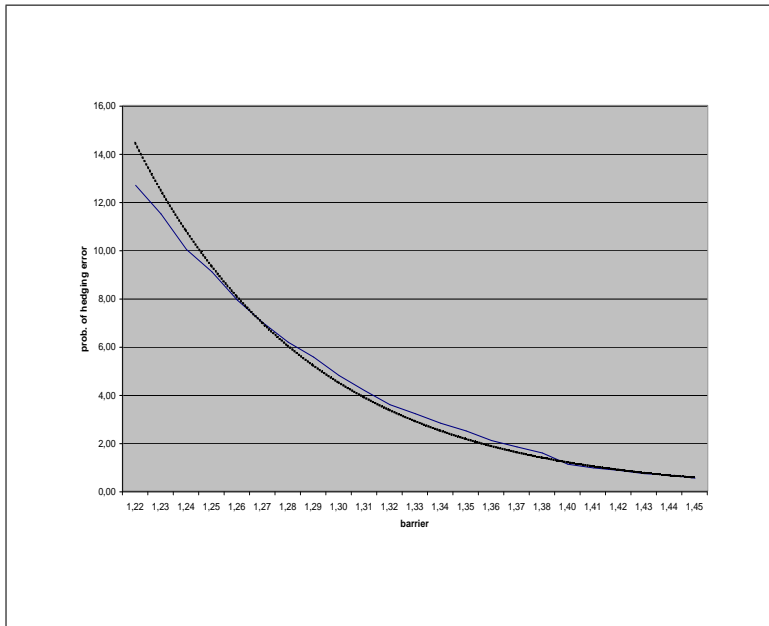


Figure 2: Probability of a miss-hedge for a discretely monitored up-and-out Call in EUR-USD

important message for the risk-averse trader. The worst case in the simulation is an error ratio of 0.08%, which drops immediately below 0.03% when we move the barrier by one big figure. With the barrier at least five or six big figures away from the spot, the error is smaller than 0.01%.

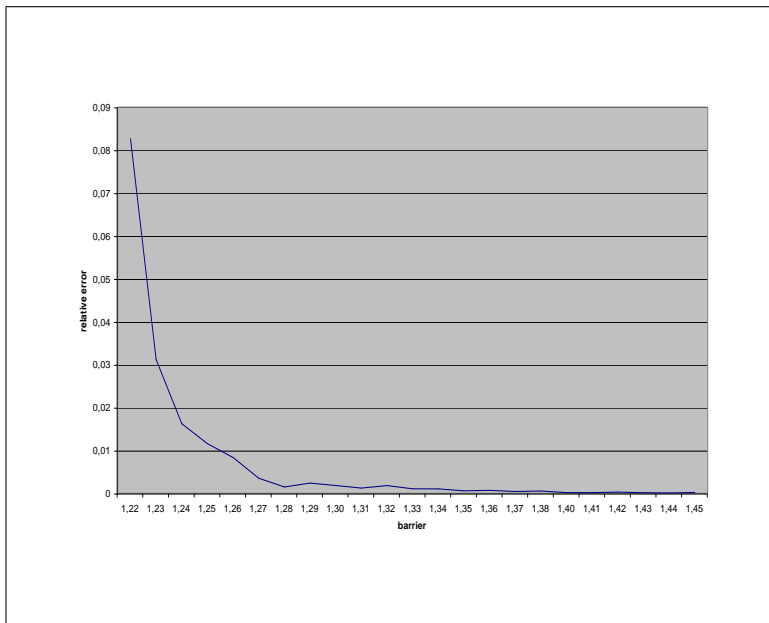


Figure 3: Hedging Error /TV for a discretely monitored up-and-out Call in EUR-USD

As the analysis of the other currency pairs is of similar nature, we list it in the appendix and continue with the conclusion.

## 4 Conclusion

We have seen that even though a trader can be in a time interval where he does not know what delta hedge he should hold for an option due to the delay of the fixing announcement, the loss encountered is comparatively small for liquid currency pairs, in which complex barrier options such as a discretely monitored up-and-out call are traded. It appears generally sufficient to charge a maximum of 0.1% of the TV to cover the potential *average* loss. This work shows that the extra premium of 10 basis points per unit of the notional of the underlying, which traders argue is needed when the underlying is the ECB-fixing instead of the spot, is not justified or at least far too big.

Of course the level of complexity of the model can be elaborated further arbitrarily, but using a geometric Brownian motion and a Monte Carlo simulation appears sufficient. The relative errors are so small that it seems reasonable not to pursue any further investigation concerning this problem.

## 5 Appendix

### 5.1 Analysis of USD-CHF

We used the market and contract data listed in Table 3.

Spot	1.2800
Strike	1.2000
Trading days	250
domestic interest rate	0.91% (CHF)
Foreign interest rate	2.17% (USD)
Time to maturity	1 year
Notional	1,000,000 USD

Table 3: USD-CHF Testing parameters

We summarize the results in Figure 4. Note that the relative error is even lower than in the EUR-USD Analysis. According to the simulation data the effect is negligible.

### 5.2 Analysis of USD-JPY

We used the market and contract data listed in Table 4.

Remarkably, in this case the curvature of the hedging error against the barrier looks different. The probability of hedging errors is higher than in the EUR-USD - case, but the important message for trading is that the ratio hedging error /

Spot	110.00
Strike	100.00
Trading days	250
domestic interest rate	0.03% (JPY)
Foreign interest rate	2.17% (USD)
Time to maturity	1 year
Notional	1,000,000 USD

Table 4: USD-JPY Testing parameters

TV reassures that practical problems can not be expected. We summarize the results in Figure 5.

### 5.3 Analysis of GBP-USD

We used the market and contract data listed in Table 5.

Spot	1.7900
Strike	1.7000
Trading days	250
domestic interest rate	2.17% (USD)
Foreign interest rate	5.17% (GBP)
Time to maturity	1 year
Notional	1,000,000 GBP

Table 5: GBP-USD Testing parameters

Again, we observe the usual picture, i.e., a similar hedging error curvature as EUR-USD, slightly higher probabilities for hedging errors, and an irrelevant practical relative error. We summarize the results in Figure 6.

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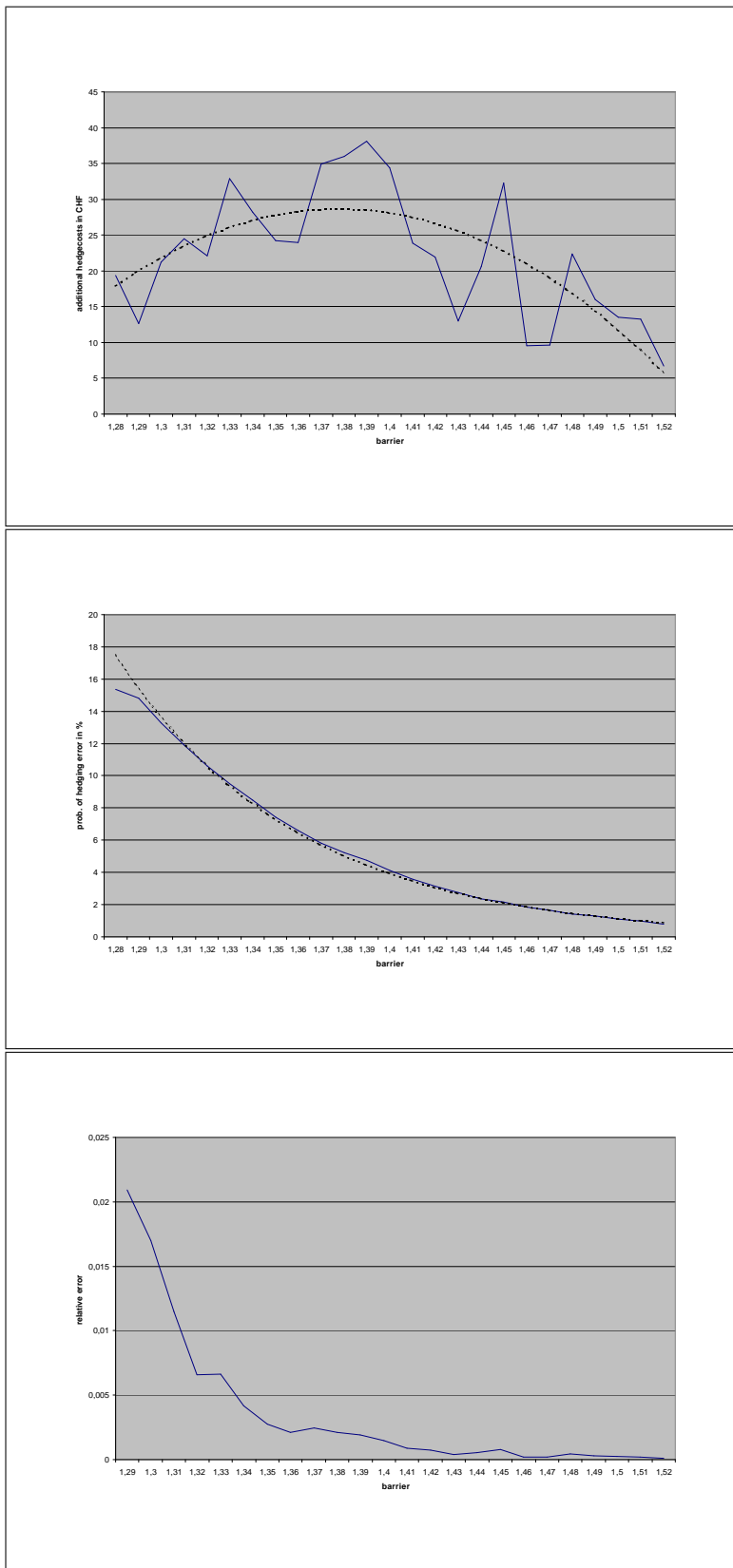


Figure 4: Analysis of the discretely monitored up-and-out Call in USD-CHF

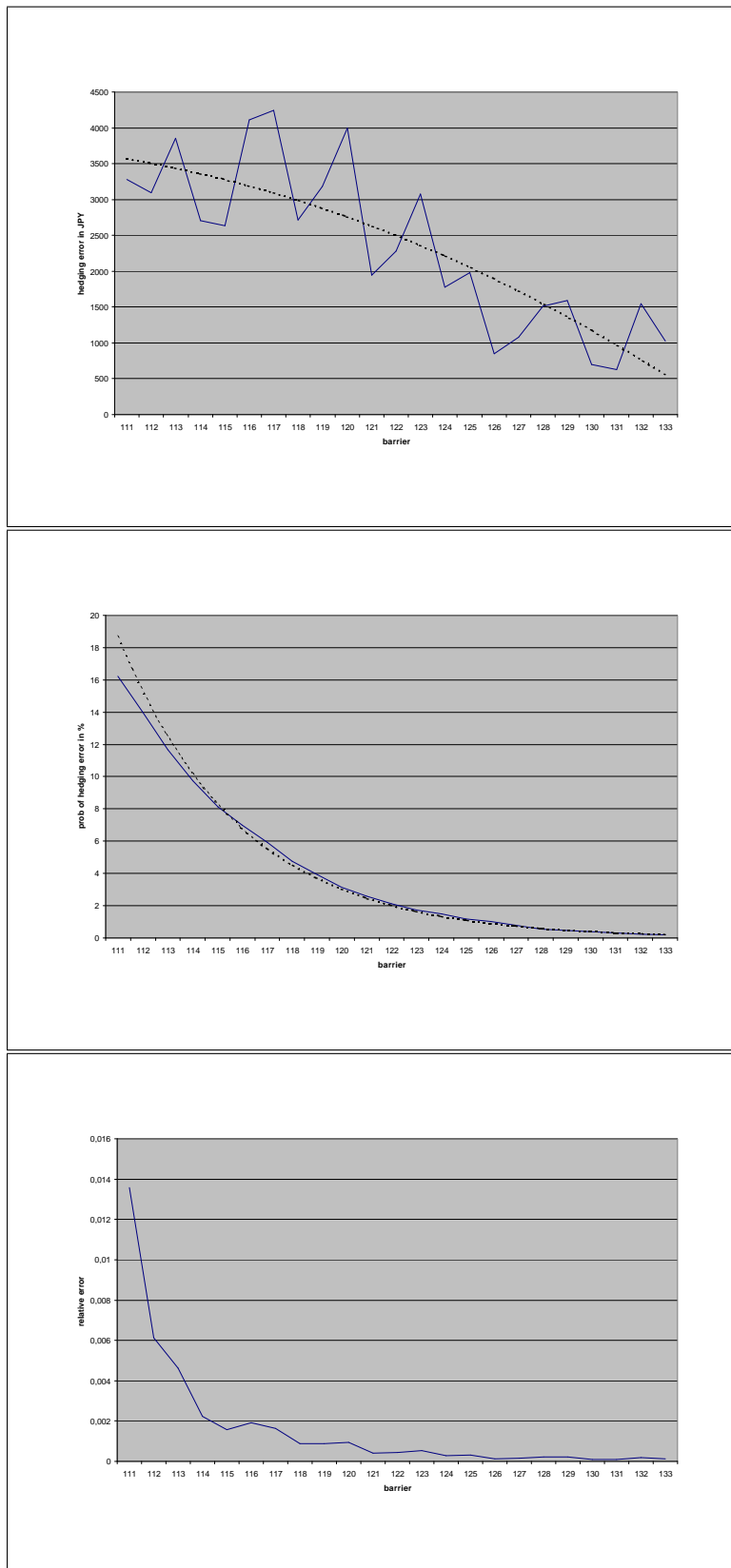


Figure 5: Analysis of the discretely monitored up-and-out Call in USD-JPY

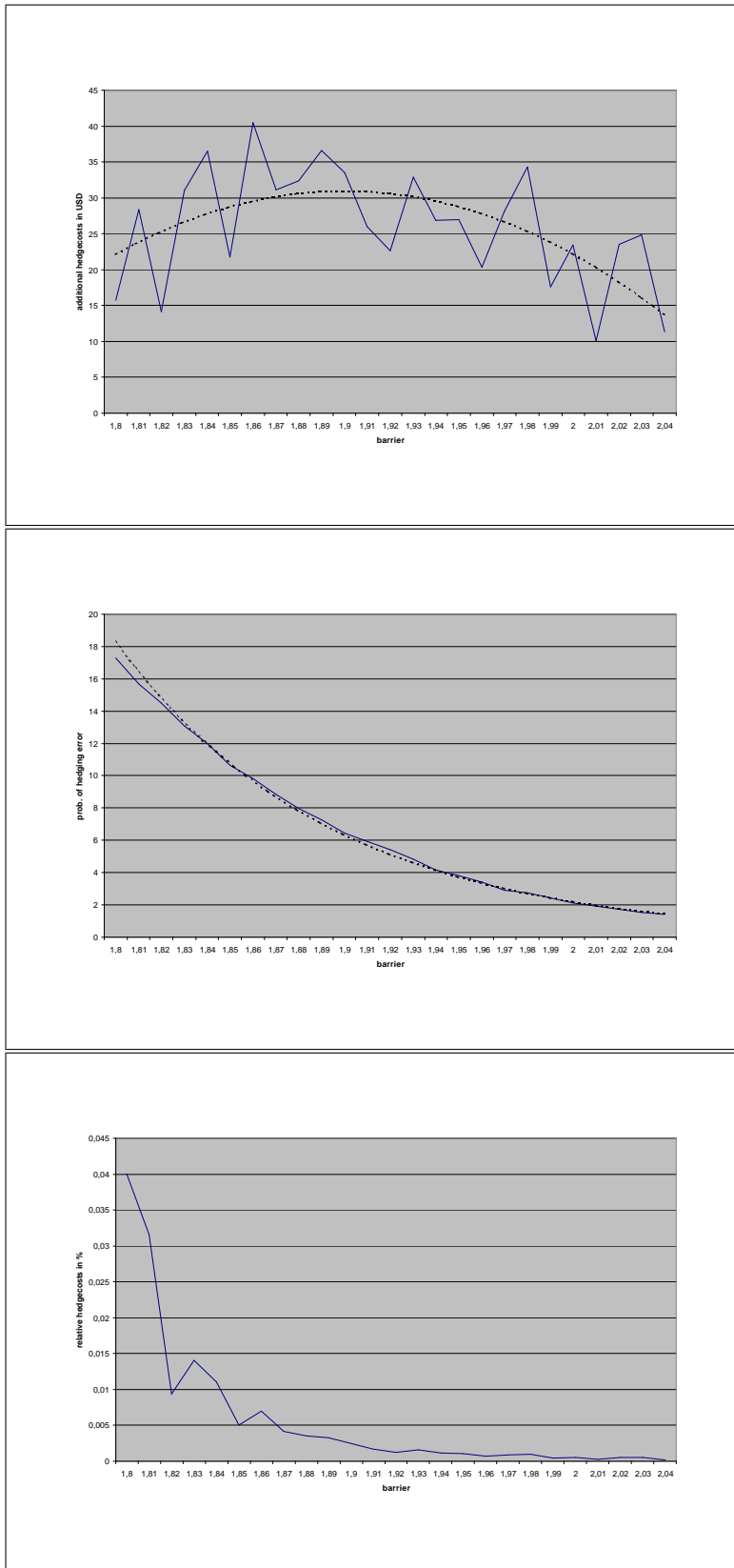


Figure 6: Analysis of the discretely monitored up-and-out Call in GBP-USD