A.I.R.A.P (Alternative Investments Risk Adjusted Performance):

Alternative Views On Alternative Investments

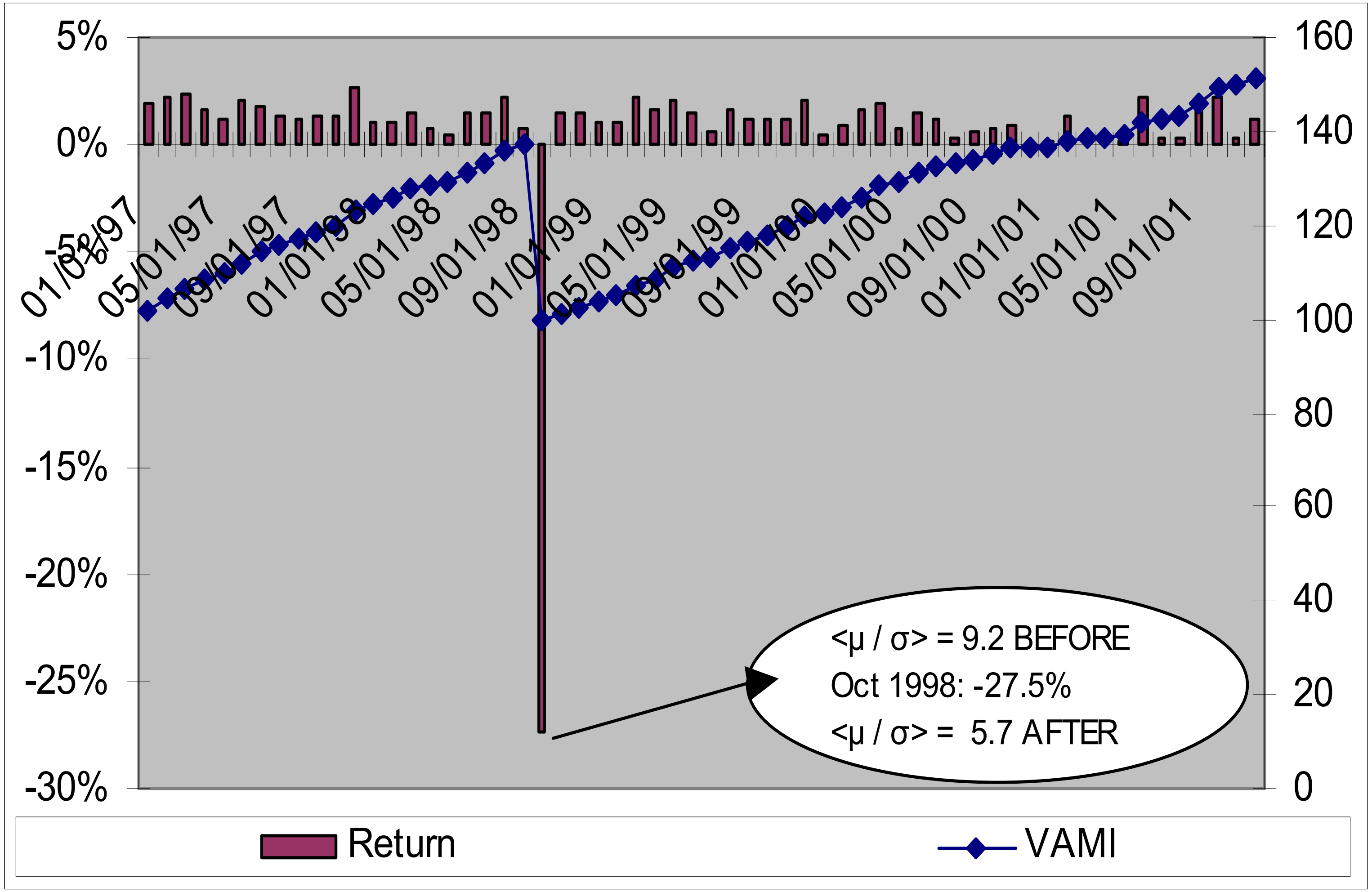
Milind Sharma*

Director

Deutsche Bank

* Opinions, errors & omissions are solely those of the author & do not represent those of employers past or present

Motivation - Short vol strategies



A legendary F.I. Arb takes a fall ...

- Could anything in the sterling track record of this hedge fund or the 9.2 Sharpe ratio have warned us of impending disaster in Oct ’98?
- How should the science of risk & performance measurement handle related phenomenon?
- Do traditional RAPMs (risk adjusted performance measures) suffice?
- Can new RAPMs like AIRAP (Alternative Investments Risk Adjusted Performance) help?
- Some of the questions I hope to address …

The PESO PROBLEM

- Characterized by smooth monthly returns punctuated by occasional catastrophic losses as we saw with FIA manager
- "*the 'peso problem' may be ubiquitous in any investment management industry that rewards high Sharpe ratio managers*" - Goetzmann et al. (2002)
- They derive the optimal static combination of short OTM puts & calls required for maximizing SR [Sharpe ratio] via truncating the right tail [i.e. left skew & fat tails]
- In general, derivatives can be structured to truncate both tails & boost SR with little impact on μ [smoothing returns]
- Bernardo and Ledoit (2000): Arbitrarily low SR possible for arbitrage opportunities while one can obtain high SR for poor investments under non-normality

The ubiquitous PESO PROBLEM

- "comparisons based on the *first two moments* of a distribution do *not take into account possible differences* among portfolios in other moments or in distributions of outcomes across states of nature that may be associated with *different levels of investor utility*." Sharpe (1994)
- How do we get a handle on this phenomenon? These so called Skew/ Kurtosis trades or trading off attractive μ & σ for higher moment risks? Or call it risk-premia earned for liquidity & event risk?
- Requires a 'sharper' RAPM which incorporates higher moments unlike Sharpe, Treynor, Alpha etc

Beyond Mean-Variance

- MV is usually justified by assuming either a] quadratic utility (arbitrary distributions) OR b] multivariate Normality (arbitrary preferences). Neither is empirically justifiable
- Levy & Markowitz (1979) argued for near sufficiency of M-V based on efficacy of 2nd order Taylor approximation for power & exponential utility
- Grinold (1999): MVO suffices for portfolio construction unless we have asymmetric distributions
- Quadratic utility: Implausible due to satiation and IARA (Increasing Absolute Risk Aversion)
- Multivariate Normality: MVN or even justification for the Central Limit Theorem is dubious once we look closer at hedge fund data given serial correlation etc

Optionality in hedge fund profiles

- Agarwal & Naik (2003): majority of HF strategies are characterized by short option profiles
- Mitchell & Pulvino (2001): Risk-Arb is akin to writing index puts. While beta neutral in up markets the strategy has high down market beta
- Fung & Hsieh (2001): CTAs can be characterized via lookback straddles [Upper Bound on perfect foresight trend following]. Tend to be long vol & event risk
- "Shorting deep out-of-the-money puts is a well known artifice employed by unscrupulous hedge-fund managers to build an impressive track record quickly" - Lo (2001)
- Whether long or short vol, most HF distributions are highly *non-normal* & there are non-linear systematic risks. Hence, linear factor models don't suffice

Challenges specific to Hedge funds

- Heterogeneity of strategies & Idiosyncratic bets
- Dynamic trading/ timing [Not amenable to linear MFMs]
- Leverage
- Shorting
- Market - Neutrality
- High turnover/ Possibly flat on close
- Significantly non-Gaussian
- Higher moment risks e.g., -ve Skew & +ve Kurtosis
- Last point is critical because σ-averse investors *prefer positive odd central moments & dislike even central moments* [cf. Scott & Horvath (1980)]

Data Issues

- HFR db: 787 surviving individual HFs (on & offshore funds + FoHFs + CTAs + sector HFs)
- 47 strategy indices: HFR & EACM are equally wt portfolios while CSFB is AUM weighted
- 5yr [01/97 - 12/01] window includes Asia, Russia, LTCM & Nasdaq crises
- Known biases: survivorship, instant history, selection etc
- Biases not consequential for relative RAPM rankings
- Survivorship correction ought to $-\mu, +\sigma$, -skew & +kurt => lower Sharpe & +divergence due to higher moments
- Approx Survivorship Bias inferred from 1-4 moments of HFR strategy indices vs. surviving funds [diversification impact embedded since indices are portfolios]

Survivorship Bias - Not just in means

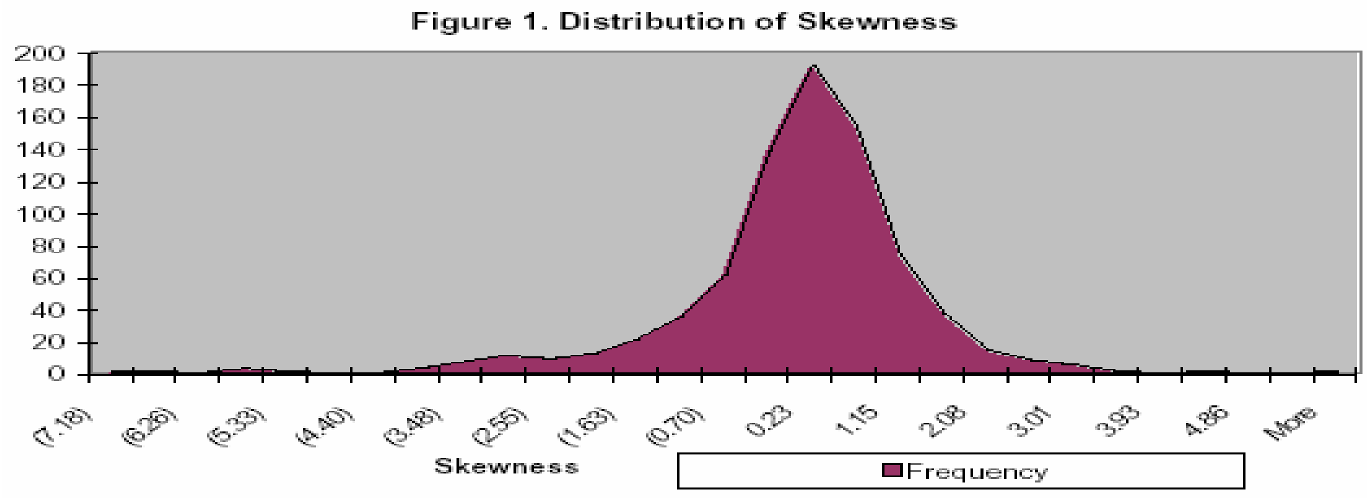
Differences	Ann mean (%)	Ann vol (%)	Skew	ExKurt	Vol style vol HFRI
Full univ	1.58	7.22	0.33	1.48	1.77
CB	−1.24	1.98	0.78	(2.29)	1.59
DS	1.75	2.98	1.57	(3.87)	1.45
EH	0.14	9.91	0.09	0.99	1.85
EMN	−0.55	5.64	(0.16)	0.58	2.49
ENH	4.35	8.33	0.37	2.01	1.47
ED	−0.12	5.92	0.77	(1.00)	1.79
FI	1.65	5.66	(0.37)	6.29	2.39
FOHF	3.06	2.04	(0.08)	1.20	1.26
Macro	2.20	8.40	0.09	1.79	2.11
RA	0.00	1.38	2.03	(8.42)	1.32
SS	1.66	6.69	(0.10)	1.22	1.24
EM	6.15	13.13	0.31	1.32	1.68
RV	2.07	1.74	2.57	(10.48)	1.44
Sector	4.47	4.99	0.04	1.32	1.25
Average	1.83	5.63	0.57	(0.67)	1.66

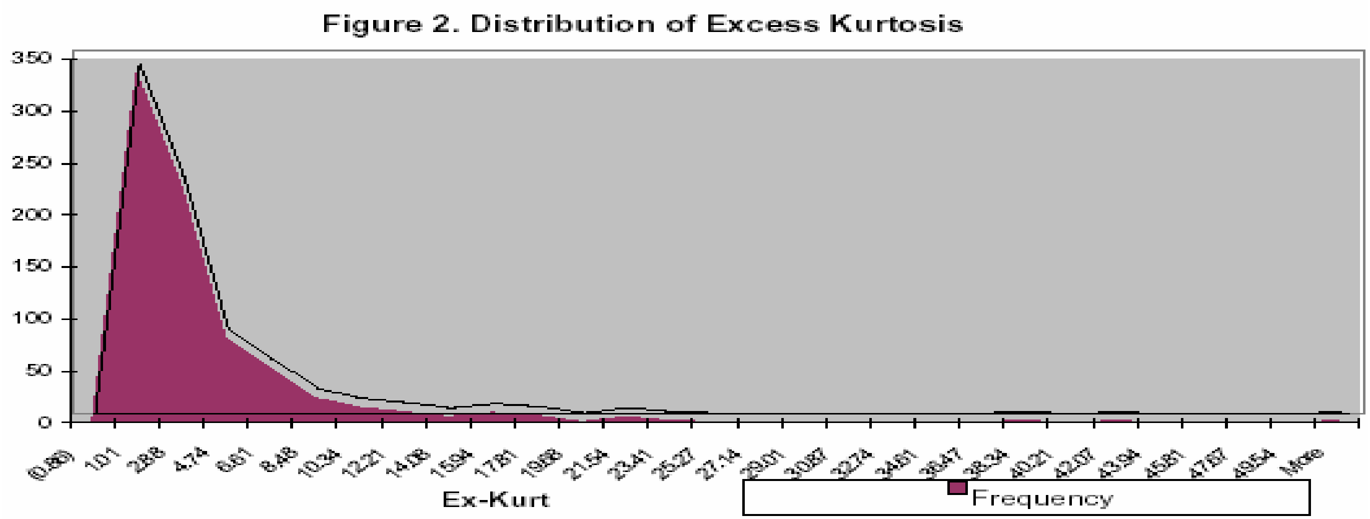
HFR indices (Table 4) are equally weighted portfolios and survivorship free [post-1994] Style means (Table 5) represent averages of individual HFR survivors intra-category. Survivor means are higher as expected with CB and EMN as notable exceptions. Survivor vols should be lower but are 1.77× higher due to diversification benefits in HFRI. Survivor skews are higher as expected but also due in part to aggregation. Survivor kurtoses are lower as expected esp. for highest kurtosis non-directional strategies, eg, RA, DS, RV and CB. Survivor kurtoses are higher in other strategies presumably due to portfolio effects. Vol style is the mean vol within style while vol HFRI is the vol of the equally weighted style portfolio.

Higher moment risks

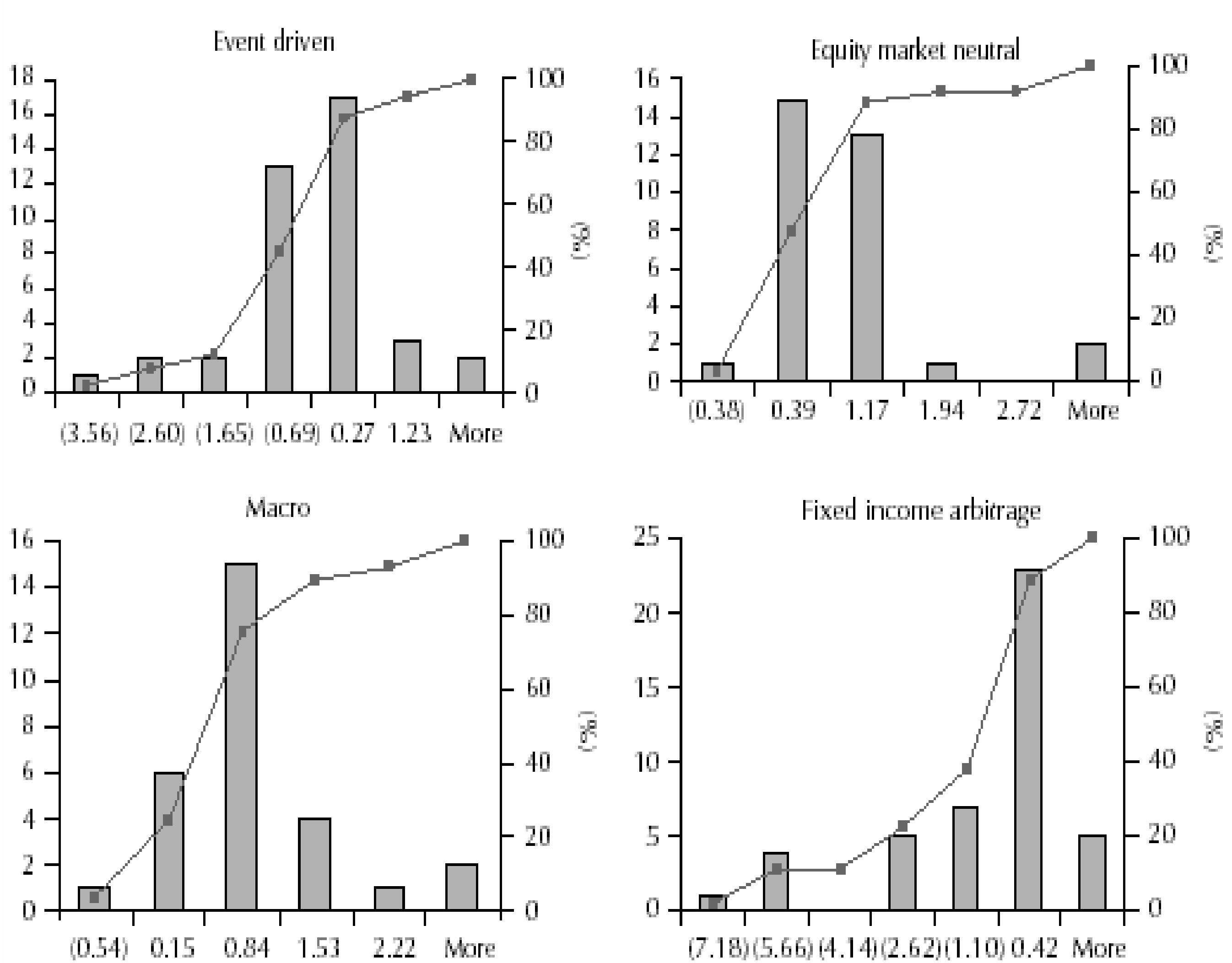
- Non-Directional strategies (RV, ED, DS, FIA, RA & CB but not EMN) have much worse -ve skew & 9.1x higher kurtoses although better $<\mu,\sigma>$ profiles => 2.4x higher Sharpe ratios
- Directional & Equity strategies [except EM] have better skew & kurt but 3.2x the σ for only 1.3x higher μ
- Macro & EMN have the most innocuous higher moments
- 8.1 average excess-kurtosis for FIA. 100% of Event Driven have excess-kurtoses > 0
- Macro & CTAs: +ve skew & often anti-correlated hence risk diversifiers

787 HFs Universe [1997-'01]

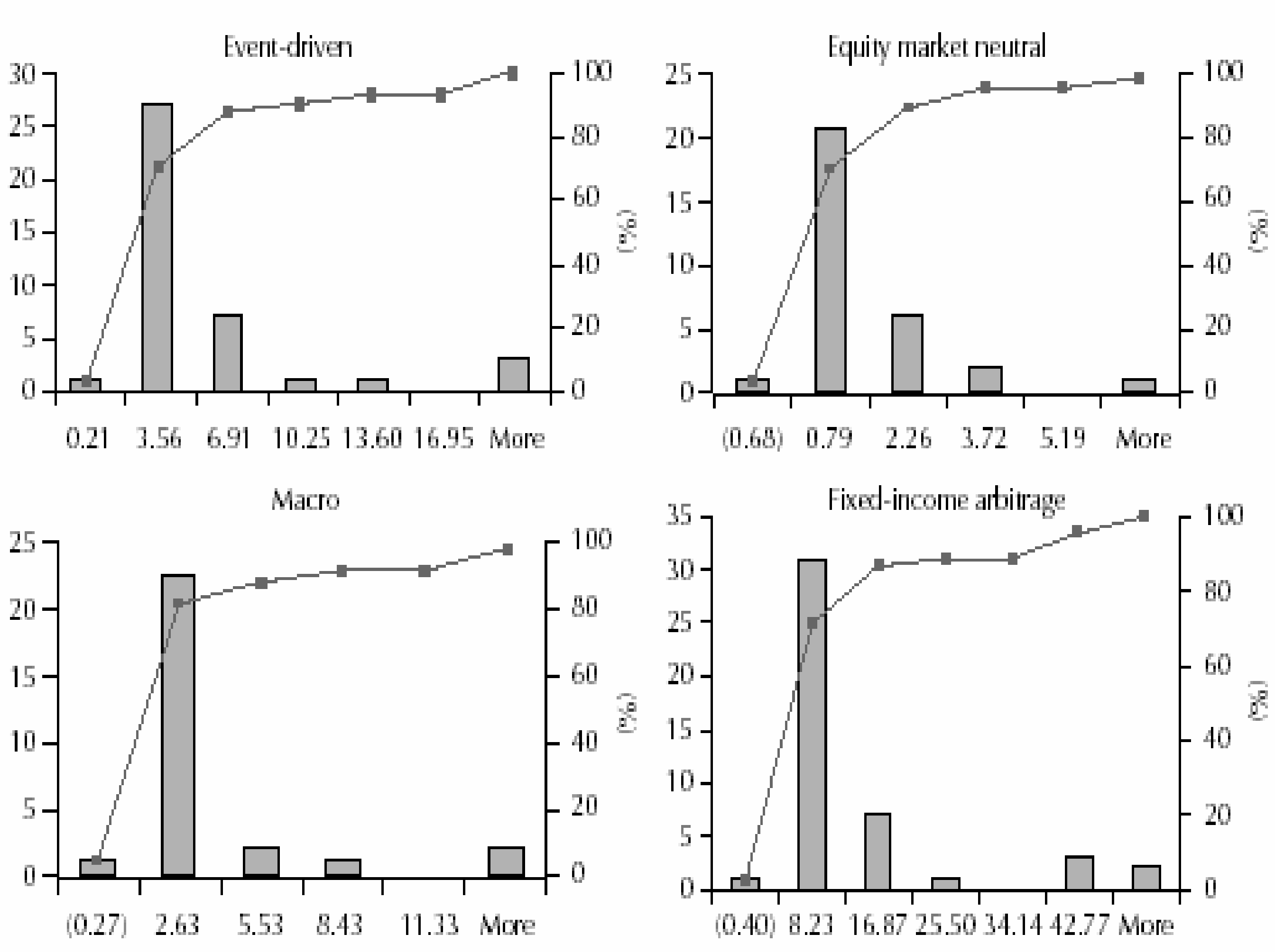




HFs Universe - Style Skews



HFs Universe -Style Excess Kurtoses



Trading off μ & σ^2 for higher moment risks?

- **Across style categories of HFR funds:**

ρ(SR, skew) = -0.64 & ρ(SR, ex-kurtosis) = +0.54

- **Across 47 HF indices:**

ρ(SR, skew) = -0.29 & ρ(SR, ex-kurtosis) = +0.25

Lower due to less survivorship which lowers SR?

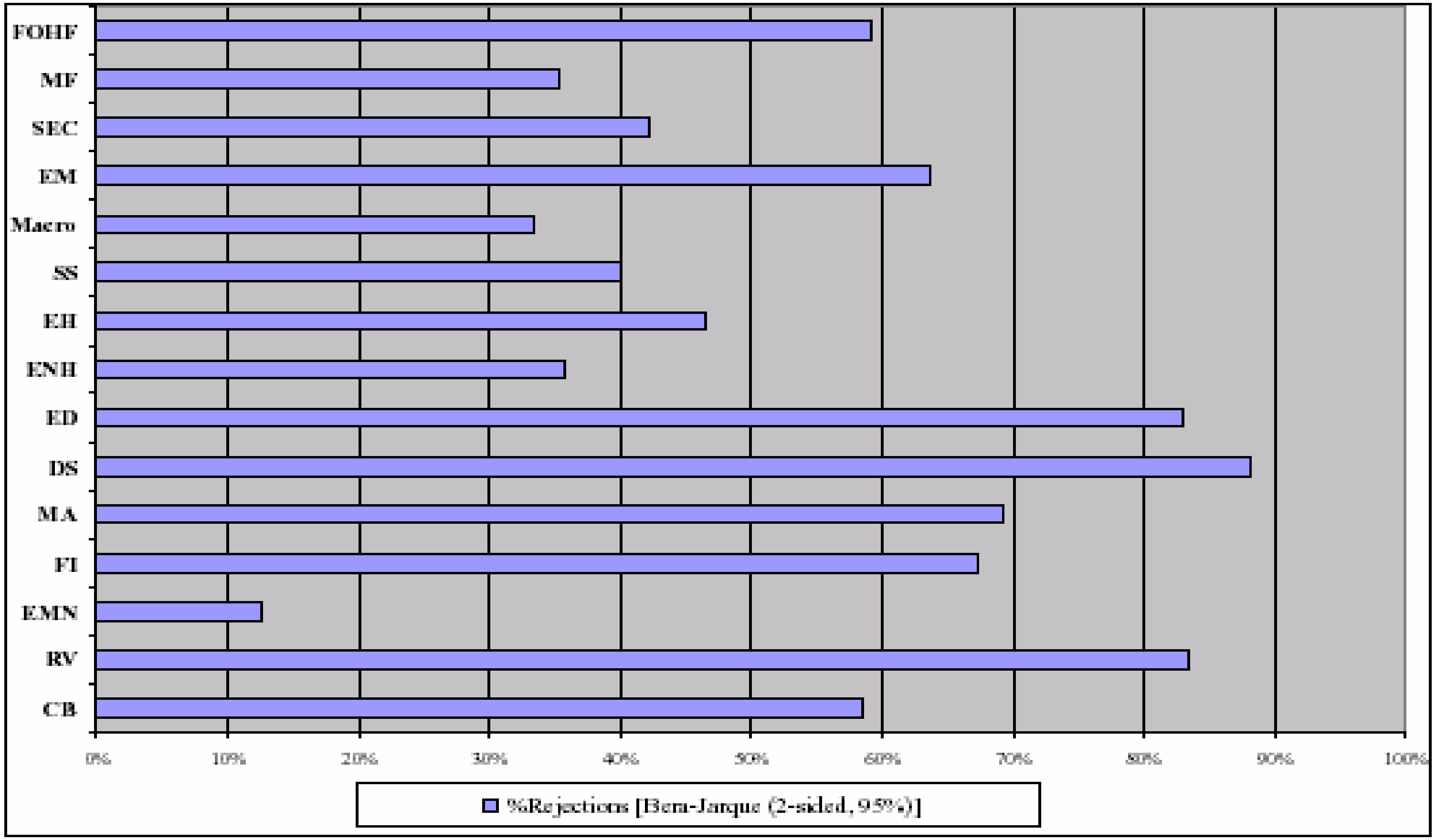
Diversification/ Aggregation effect?

- **Across HFR universe (all funds of all styles):**

ρ(SR, skew) = +0.07 & ρ(SR, ex- kurtosis) = +0.12

- Essentially washes out … but not accounting for it in style optimization & simply maximizing SR can lead to maximizing higher moment risks & illiquidity

%Rejections of Normality



Highest rates of deviation from Normality (Bera-Jarque GoF test) occur in the **least liquid & most volatile strategies**

RAPMs menagerie

- Sharpe, Jensen, Treynor, M^2, M^3, Information, Sortino, Calmar, Sterling, Gain/ Loss, SHARAD (Skill, History & Risk-Adjusted) + modified & generalized versions
- Performance statistics - Max Drawdown, Time to recovery, Peak-Trough, VAMI, up/ down returns, upside/ downside capture etc
- Risk metrics - Beta, active/ total risk, variance, semi-variance (upside/ downside & capture), LPMs, VaR, VarDelta, Marginal VaR, CVaR [Expected Shortfall] etc
- What suffices under non-normality?

Risk of RAPM shortfall

- Treynor – Unbounded esp. for beta neutrality (EMN), sign problem for -ve beta exposures (SS, CB, CTAs)
- Alpha – Leverage proportionality; Can be misleading for Shorts & Non-directional strategies
- SR – Leverage invariant; can be 'gamed' under non-normality; oblivious to preferences & higher moments; symmetric risk penalty
- Sortino & Downside Deviation measures – Underestimate if upward trend during in-sample period followed by reversal out-sample

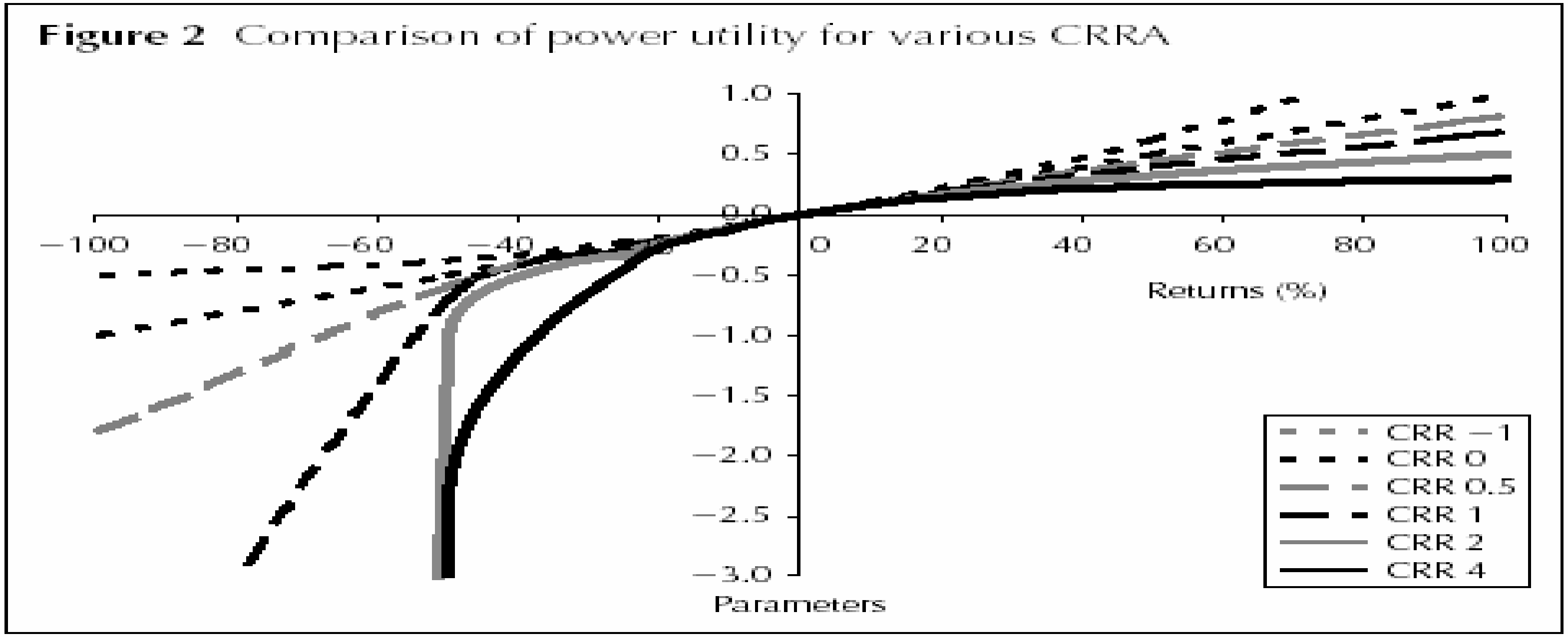
Alternatives?

- Modified Sharpe ratios: Incorporate higher moments in denominator via Cornish-Fisher V@R, t-dist V@R or utility based risk-premium (AIRAP RP). May inherit problems from SR e.g. case of –ve means
- Modified CAPM Beta (ass. Only IID market) - Leland (1999)
- Gain/ Loss ratios
- Certainty Equivalent: CARA case in Madan & McPhail (2000)

AIRAP

- AIRAP is CE under CRRA with default value of CRRA=4
- Arrow-Pratt RRA = $-\frac{u''(w)}{u'(w)} w = c$
- All we need assume is risk-aversion & more is better than less [u'>0, u''<0 & u is C^2], where c>0

Power Utility family



$$U(W_T) \equiv \frac{W_T^{(1-c)} - 1}{(1-c)}, c \neq 1, c \geq 0 \; and \; U(W_T) \equiv \ln(W_T) \; when \; c = 1$$

$$U(1+\mathrm{TR}) \equiv \frac{(1+TR)^{(1-c)} - 1}{(1-c)}, c \neq 1, c \geq 0 \; and \; U(1+\mathrm{TR}) \equiv \ln(1+TR) \; when \; c = 1$$

Use scale invariance to express in total returns instead of W(T)

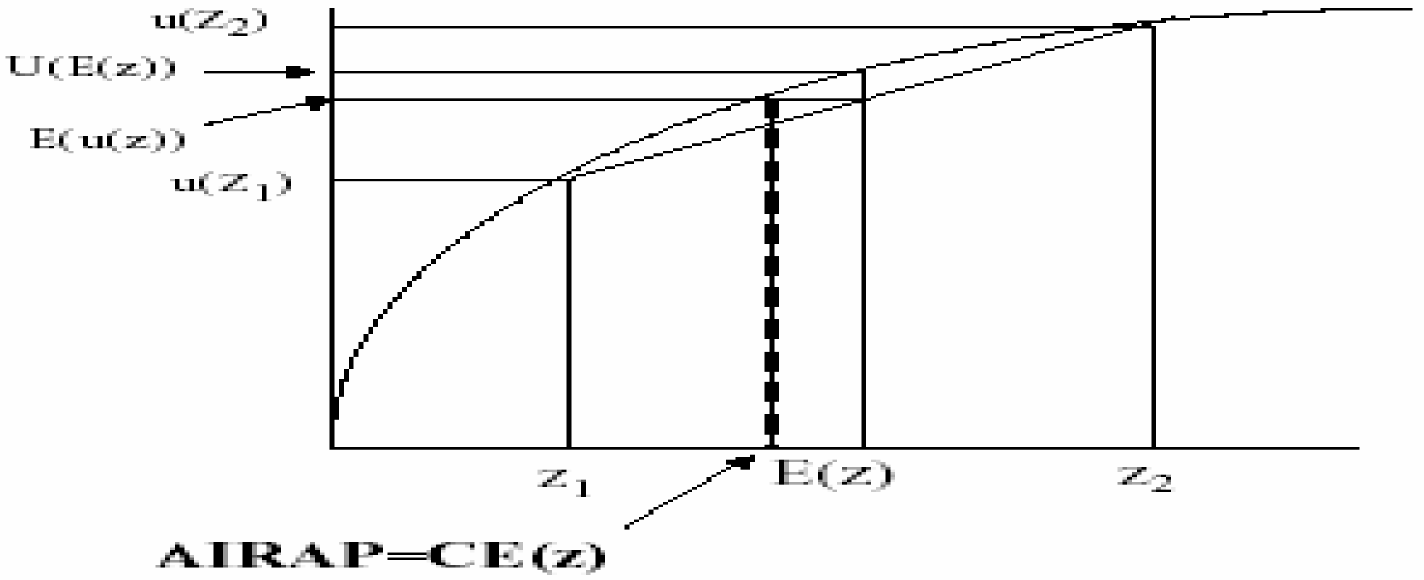
AIRAP derivation

Given VNM axioms, preferences have EU representation. Risk aversion is embodied by the concavity of *u* (risk-proclivity by convexity and risk-neutrality by linearity). Desired in util fn a] u'>0 b] u"<0 c] non-IARA

Concavity & Jensen's inequality yield: ***u*(E) > E(*u*)**

Let **CE(z) := *u*$^{-1}$(E(*u(z)*))** such that the payoff **CE(z) < E(z)**

Hence, the Risk-Premium can be defined as, $RP(z) = [E(z) - CE(z)]$



AIRAP derivation

Given observed discrete distribution solve from **u(CE(z)) = E(*u(z)*):**

$$EU \equiv \sum_i \left[\frac{(1+TR_i)^{(1-c)} - 1}{(1-c)} \right] . p_i = U \Leftrightarrow \left[\sum_i p_i .(1+TR_i)^{(1-c)} - \sum_i p_i \right] = (1+CE)^{(1-c)} - 1$$

$$\Leftrightarrow \left[\sum_i p_i .(1+TR_i)^{(1-c)} \right] = (1+CE)^{(1-c)}$$

$$AIRAP = CE = \left[\sum_i p_i .(1+TR_i)^{(1-c)} \right]^{1/(1-c)} - 1, \quad when \quad c \neq 1 \,\&\, c \geq 0$$

$$EU \equiv \sum_i \ln(1+TR_i).p_i = U \equiv \ln(1+CE), c = 1$$

$$\Leftrightarrow \ln \left\{ \prod_i (1+TR_i)^{p_i} \right\} = \ln(1+CE)$$

$$\Leftrightarrow AIRAP = CE = \left[\prod_i (1+TR_i)^{p_i} \right] - 1, \quad c = 1$$

AIRAP (Certainty Equivalent for CRRA=4)

$$\text{AIRAP} = \text{CE} = \left[\sum_i p_i.(1+\text{TR}_i)^{(1-c)}\right]^{\frac{1}{(1-c)}} - 1, \quad \text{when} \quad c \neq 1 \ \& \ c \geq 0$$

$$\text{AIRAP} = \left[\prod_i (1+\text{TR}_i)^{p_i}\right] - 1, \quad \text{when} \quad c = 1$$

- Use any non-parametric fit in above
- OR fit the degenerate histogram for special case of closed form where ∀ non-empty bins, $p_i = 1/N$ as a result of setting the bin widths as,
- $\varepsilon := ½*\text{Min}\{|\text{TR}_i - \text{TR}_j|\}, \forall\ i \neq j$
- Starting with the leftmost observation, the ε-bins are centered on each TR_i such that all distinct TR_i fall in exactly one bin
- If k identical returns are actualized then assign k/N for that bin
- Taking the extreme case for histogram eliminates arbitrariness in precision from arbitrariness in bin width

Arrow-Pratt coefficient

•**c>0** represents risk-aversion since u'>0 => u''<0. AIRAP resembles the L_p norm, for p=(1-c), except that $p \neq 0 \,\&\, p \leq 1$

•**c=0** => u''=0 so U(TR) is linear in %TR & AIRAP is simply the arithmetic mean or in the annualized case it is the compound monthly arithmetic mean excess return

•**c=1**, log utility case results in AIRAP as the geometric mean of monthly excess returns

•**0<c<1** implausibly allows bets potentially resulting in insolvency

•**c≥1** This restriction keeps us out of bankruptcy court & is consistent with the limited liability fund structure

Arrow-Pratt coefficient

- Ait-Sahalia et al. (2001): c=3.2 (s.e. 2.2) from data on HNW consumption of luxury goods while c=4.7 (s.e. 3.3) is implied by ‘Charitable Contributions of Rich’. Can help resolve equity premium puzzle if true that NIPA & household survey of basic goods consumption overstates risk aversion by an order of magnitude
- Osband (2002) recommends c in [2,4] range
- Default used: c=4 to be conservative

Preferences matter: Value added is a function of CRRA

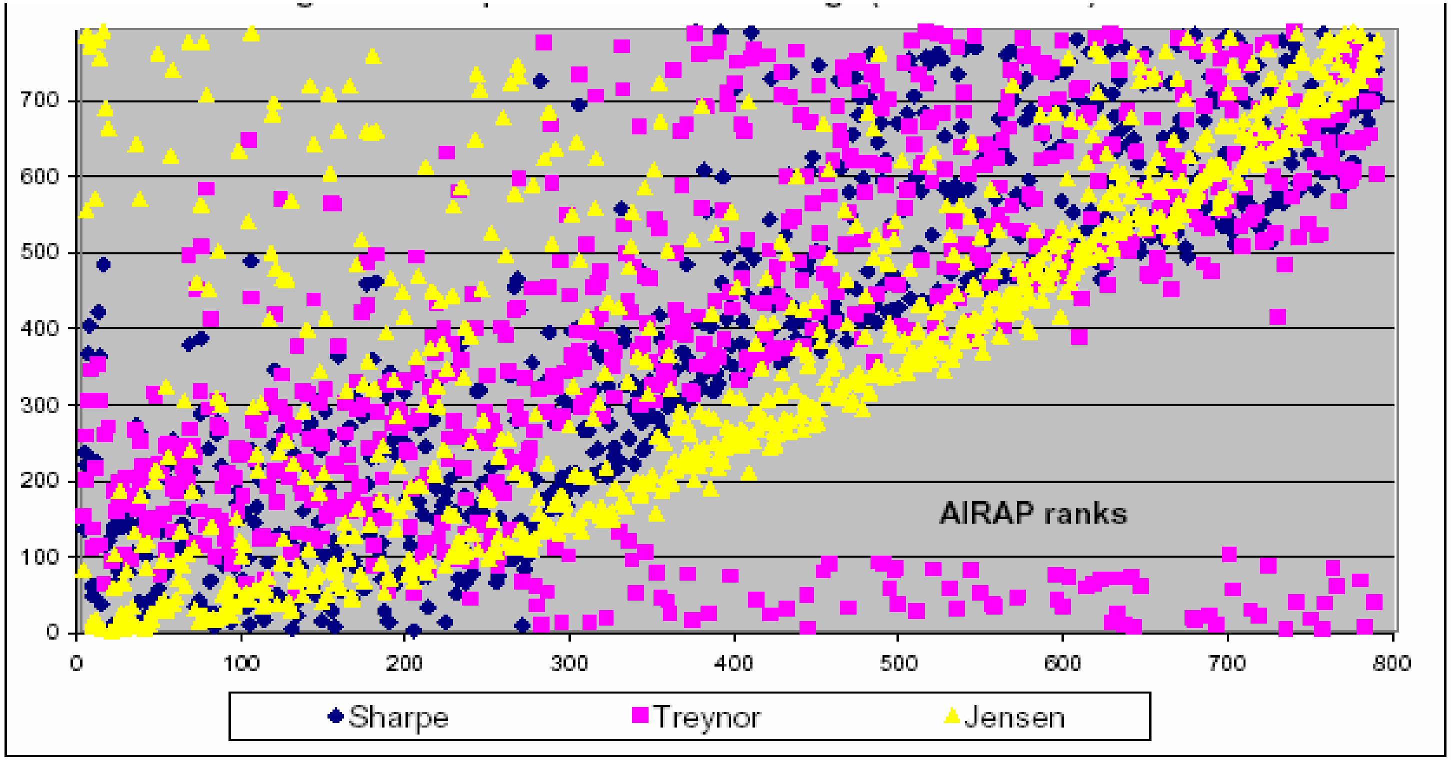
CRR	No. of funds [AIRAP > 0]	% [AIRAP > 0]
0.1	709	90
0.2	704	89
0.3	698	89
0.4	695	88
0.5	690	88
1	662	84
1.5	641	81
2	620	79
2.5	597	76
3	565	72
3.5	538	68
4	514	65
4.5	498	63
5	475	60
10	333	42
15	260	33
20	225	29
25	183	23
30	163	21

% [AIRAP > 0] is a proxy for value added by hedge funds.

AIRAP highlights

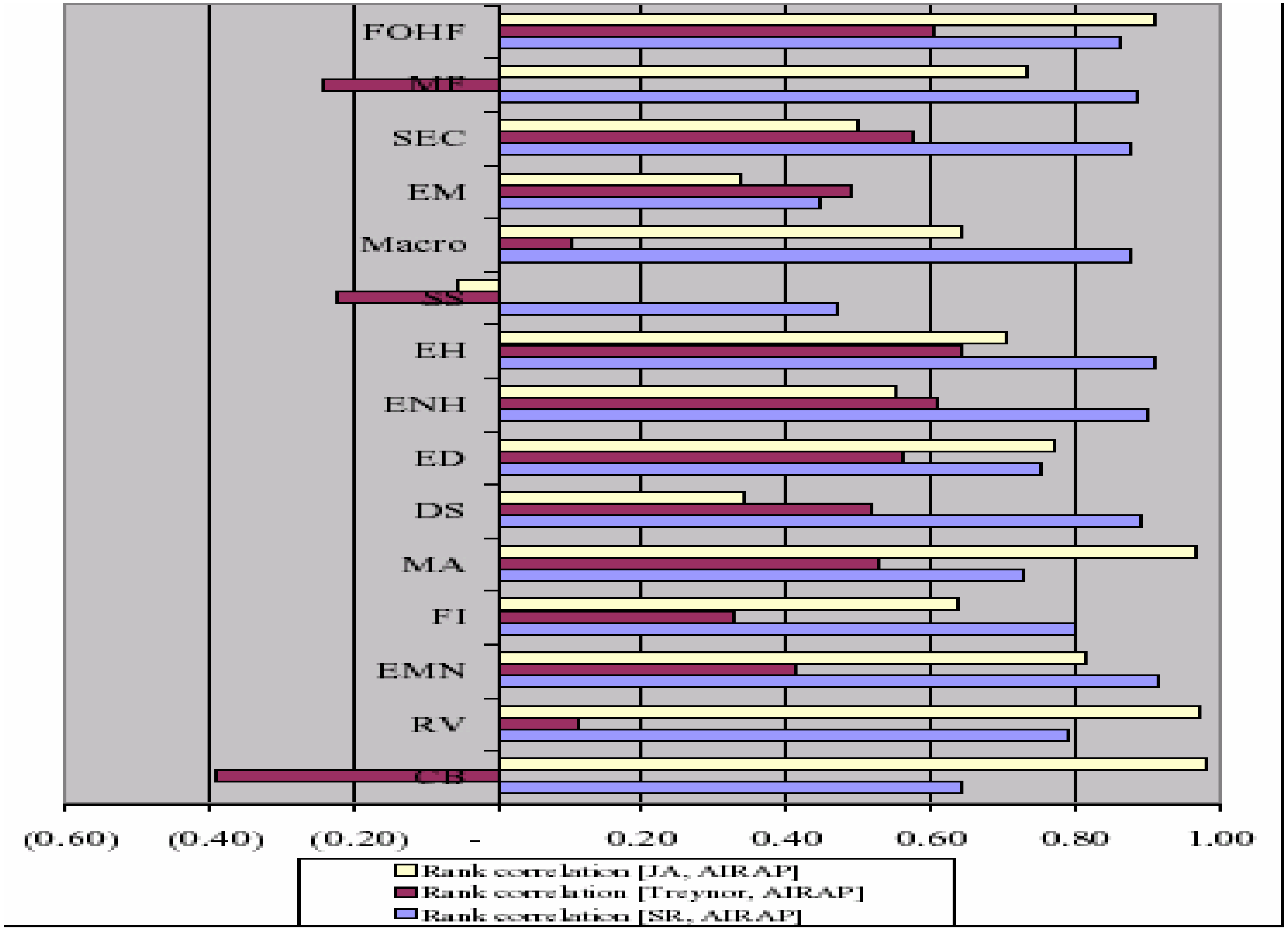
- Distribution free approach
- Adjusts for leverage - unique
- Downside variance is penalized more
- All higher moments are used (no truncation/ no convergence issues)
- Works for mean excess returns < 0
- Equivalent to maximizing EU for portfolio construction of FoHFs or multi-strategy HFs
- Combine with canned scenario stresses for handling iceberg risks ex-ante
- Consistent with LLC & fiduciary impartiality
- Straightforward spreadsheet implementation

RAPM Rankings vs. AIRAP



All RAPM ranks are in ascending order with higher ranks being more desirable
AIRAP (CRRA=4) rankings are shown on the x-axis
The abundance of off-diagonal data shows the extent of divergence between the 3 RAPMS vis-à-vis AIRAP
The cluster of pyramids in the top left represents high JA funds demoted by AIRAP
The cluster of squares at the bottom right represents high AIRAP funds demoted by Treynor
Source for 787 HFs used is Hedge Fund Research, Inc., © HFR, Inc., www.hedgefundresearch.com

Spearman correlations by Style



Rank correlations with AIRAP

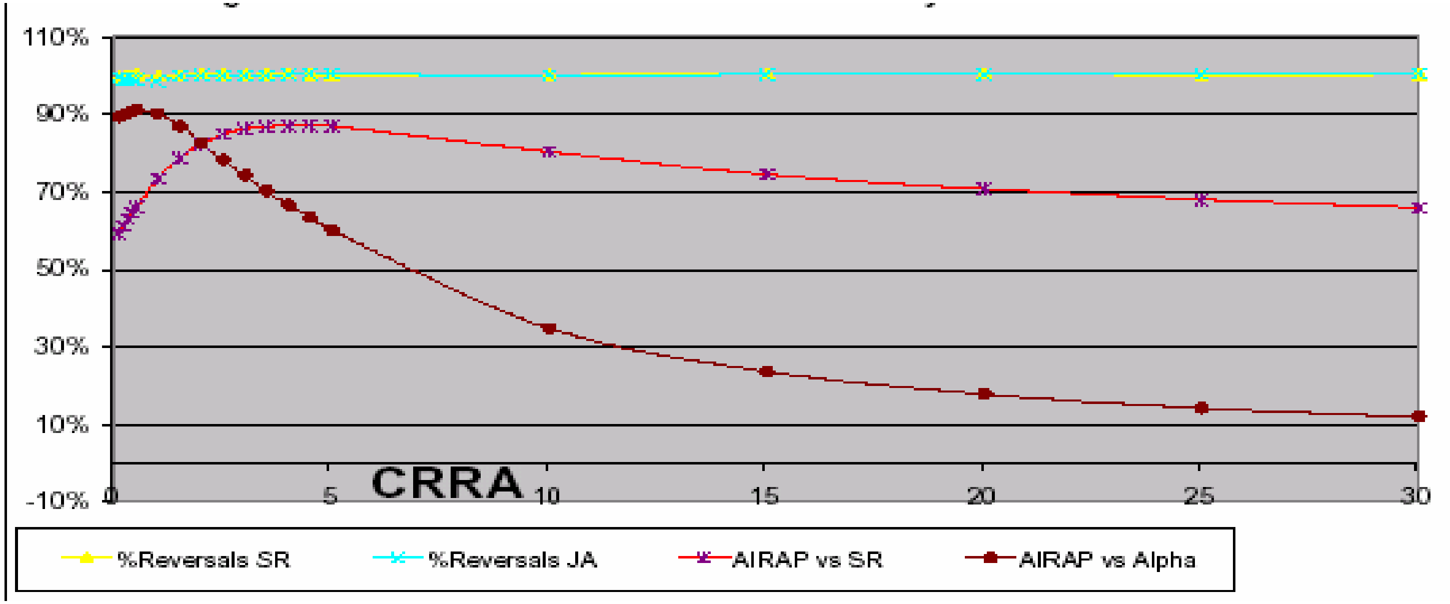
- Non-Directionals (RV, MA, ED and CB) with greatest higher moment risks ➔ Lowest correlations with Alpha & Treynor
- High σ penalty (SS & EM): Lowest correlations w/ SR, JA & Treynor
- [β<0] cases SS, MF & CB: [ρ<0] with Treynor
- Directionals with innocuous higher moments viz., EMN, EH, ENH, Macro, SEC & MF: SR high correlation (>0.87)
- Impact of vol (2nd moment) often dominates that of 3rd & 4th

AIRAP vs. SR & Alpha by CRRA

CRR	#Reversals SR*	#Reversals JA*	%Reversals SR	%Reversals JA	AIRAP vs SR	AIRAP vs Alpha
0.1	781	779	99%	99%	0.59	0.89
0.2	780	775	99%	98%	0.61	0.89
0.3	784	777	99%	99%	0.63	0.90
0.4	783	776	99%	98%	0.64	0.90
0.5	786	776	100%	98%	0.66	0.91
1	783	772	99%	98%	0.73	0.90
1.5	784	783	99%	99%	0.78	0.86
2	787	785	100%	100%	0.82	0.82
2.5	785	784	100%	99%	0.84	0.78
3	785	784	100%	99%	0.86	0.74
3.5	786	783	100%	99%	0.86	0.70
4	787	786	100%	100%	0.86	0.66
4.5	786	786	100%	100%	0.87	0.63
5	783	787	99%	100%	0.87	0.60
10	785	784	100%	99%	0.80	0.34
15	788	786	100%	100%	0.74	0.23
20	788	786	100%	100%	0.70	0.17
25	784	787	99%	100%	0.68	0.14
30	786	787	100%	100%	0.66	0.11

AIRAP vs Sharpe & Jensen represent Spearman rank correlations
Correlation with Jensen tapers off rapidly

AIRAP vs. SR & Alpha by CRRA



•Rank correlations with Alpha taper off quickly for CRRA > 1 & drop dramatically in the [1,10] range

•Rank correlations with Sharpe peak at CRRA=5 then decline

•While SR correlations are not particularly low in the [1,10] range, the divergent cases are extreme enough to merit attention. Can one afford fewer but costlier errors in portfolio construction?

Alpha - perverse incentive to lever up

- Alpha overstates RAP by 7.2% on average vs. AIRAP esp. for Non-Directional strategies or where CAPM beta misses other sources of systematic risk
- Alpha misleading for Shorts OR is perhaps AIRAP vol penalty too steep? (Alpha - AIRAP) = 37.1%
- Alpha scales up with leverage since μ, σ & β do
- Sharpe: When $\mu<0$, twice the vol has higher Sharpe

Ann vol	6.02%	12.04%
Ann %ExTR	-4.50%	-4.50%
Ann Sharpe	(0.75)	(0.37)

HF Case studies

- #512: Worst AIRAP rank (1) but SR rank is 133 (of 787), because not only are returns low (10) & 56% Vol extreme (776) but iceberg risks are high (ExKurt = 682 & Skew = 58)
- #235: Worst SR (1) due to -ve mean & low Vol (8). AIRAP (202) correctly boosts rank +201 given low Vol & tame higher moments
- #229: Highest Alpha (787) due to -ve beta & due to highest ExTR(787) , but middle of the pack SR (482) since vol (787) also highest, while AIRAP (13) is 774 notches lower because of penalty for extreme 83.2% Vol. Skew is favorable, kurtosis is moderately high

Representative fund RAPM comparisons

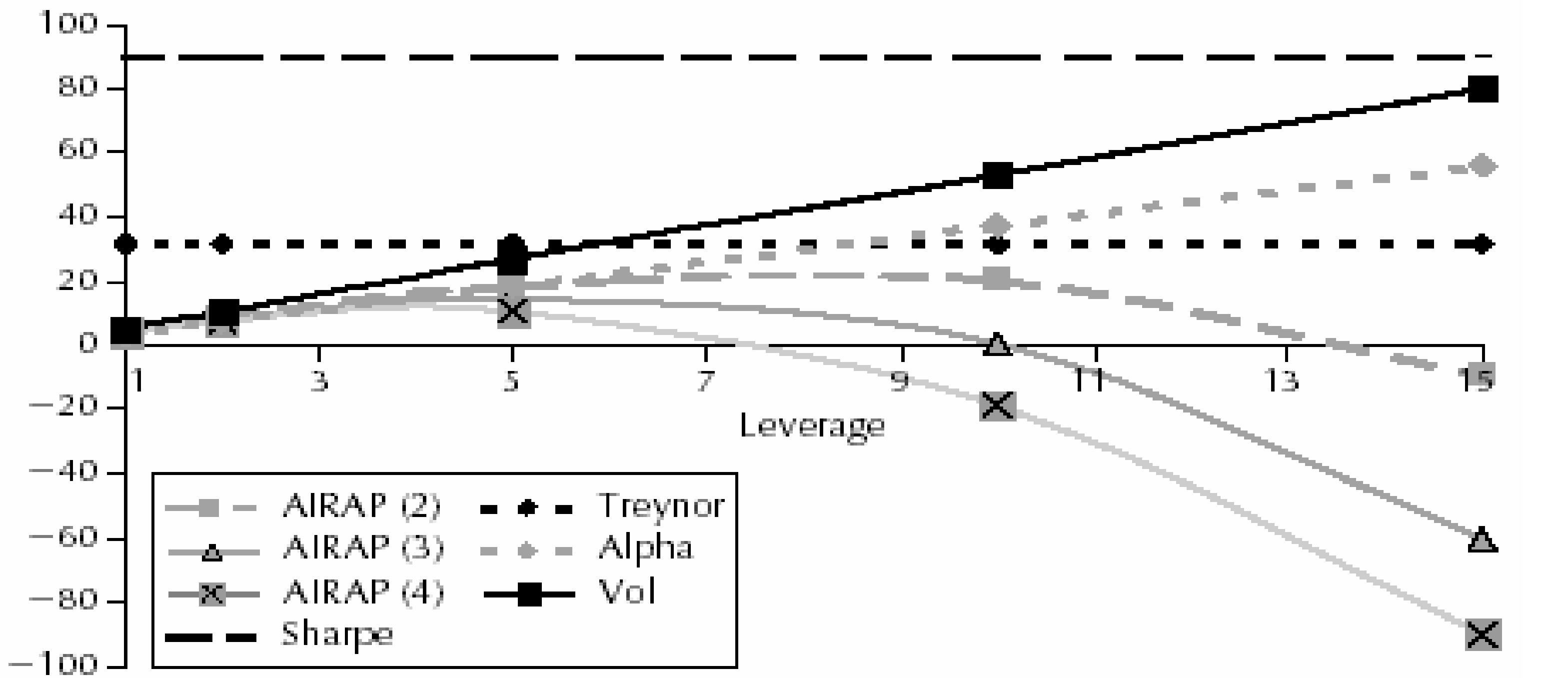
Fund ID	AIRAP	Sharpe	Treynor	Jensen	Beta	ExTR	Vol	Skew	ExKurt
229	**13**	482	62	**788**	25	787	**787**	719	590
230	11	362	302	753	786	679	**781**	398	507
231	10	420	351	777	783	747	**780**	373	550
235	202	**1**	48	65	176	69	8	423	74
272	**2**	234	256	**781**	655	223	**788**	**788**	**786**
512	**1**	133	151	81	771	**10**	**776**	58	**682**
636	**788**	699	599	**776**	635	784	545	667	361
762	373	**788**	784	221	143	201	2	485	174

Fund ID	AIRAP	Sharpe	Treynor	Jensen	Beta	ExTR	Vol	Skew mo	ExKurt
229	-48.57%	0.76	(1.27)	66.53%	(0.50)	37.92%	83.16%	1.13	3.23
230	-51.15%	0.53	0.19	20.27%	1.75	14.09%	61.08%	0.01	2.27
231	-51.64%	0.62	0.23	25.93%	1.64	20.12%	60.23%	(0.05)	2.68
235	-2.76%	-1.72	(1.40)	-2.88%	0.02	-2.72%	1.60%	0.07	(0.10)
272	-86.14%	0.34	0.49	29.23%	0.69	3.03%	100.07%	5.78	41.15
512	-93.25%	0.15	0.06	-2.09%	1.45	-13.39%	56.17%	(1.82)	6.00
636	25.63%	1.54	0.48	25.40%	0.62	31.96%	19.32%	0.80	1.13
762	2.41%	7.54	9.38	2.37%	0.00	2.41%	0.32%	0.22	0.34

Leverage * – Perils & Optimality

- Critical extra degree of freedom – needs adjustment
- Brown, Goetzmann & Park (2001): Watch delta of incentive option. Agon between reputational costs & survival vs. swinging for the fences
- Kritzman & Rich (2002): Dramatic rise in within-horizon (vs. terminal) 10% DD probabilities
- Leverage invariance (Sharpe/ Treynor) desirable for traditional investments but not for HFs
- Alpha: perverse incentive to lever up (FIA, relative-value strategies)
- Leverage optimality from maximizing AIRAP
- *External funding leverage

Leverage optimality



CRRA/ Leverage	1	2	5	10	15
1.00	4.77%	9.44%	22.63%	39.85%	45.23%
2.00	4.62%	8.84%	18.48%	20.32%	-9.44%
3.00	4.48%	8.24%	14.39%	0.86%	-60.10%
4.00	4.33%	7.64%	10.30%	-18.89%	-90.27%
10.00	3.47%	4.07%	-15.83%	-93.48%	-100.00%

Leverage - criticality

- Critical to HF investing yet ignored
- Other RAPMs do not explicitly account for leverage
- AIRAP allows us to infer the optimal leverage consistent with investor risk aversion and the risk signature of the strategy or the individual HF

Further research/ refinements

- Adjust for serial correlation first e.g., sum of lagged betas in Asness, Krail & Liew (2001), Dimson (1979) or Scholes & Williams (1977)
- Use of CE in imputing unsmoothed returns?
- Avoid annualization by root 12: Lo (2002) showed this can result in upto 65% overstatement of Sharpe ratios
- Careful of the plausibility of downside vol penalty
- Incorporate correlations/ marginal contributions in Fo]hF context
- Try disappointment aversion/ prospect theory
- Contrast FoHF portfolio allocations from Max AIRAP vs. Max SR [see e.g., Hagelin & Pramborg (2005)]

Concluding thoughts

- Are RAPMs useful in a forward looking sense? Not as used in practice - They tend to understate ex-ante risks at the top of the cycle & overstate risks at market troughs
- Icerberg risks & rare events are not likely to show up in the short history available for HFs. Hence backfill history via style signature as per modification of Sharpe(1992) e.g., Agarwal & Naik (1999) or Fung & Hsieh (1998)

<u>Peer %tile composite ranking</u>

- *Composite AIRAP* %tile = {w_1**AIRAP Style* %tile + w_2**AIRAP Stress* %tile}, where
- *AIRAP Style* %tile = 5y AIRAP %tile ranking within style category

References

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