On the question of how to measure future exposure of capital market transactions in the context of counterparty risk modelling under Basel II

Discussion of expected positive exposure (EPE) approach

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Dresdner Bank
Advice you can bank on

Motivation

The task – portfolio loss calculation

- Have a portfolio of counterparties (the "portfolio").
- Each counterparty has a portfolio of derivatives (the "counterparty portfolio").
- Task: To investigate the distribution of losses from the portfolio.
- Has much in common with loan case, but with some interesting extra ideas.

Motivation 1 – Basel II regulatory capital

- Current capital calculation for counterparty risk is broadly (Mark-to-Market + Add-on) $\times$ 4% (where for Corporate Counterparts 4%=50%*8%).
- Add-ons based on work done by regulators in 1995 / 6.
- Basel II IRB approach will use IRB risk weights instead of 4%, but same old add-ons.
- The work described here arose from ISDA's attempts to find a risk sensitive alternative calculation more in keeping with IRB approach generally.

Part 1

Introduction to the problem of how to measure future counterparty exposure

Basel II Regulatory Capital

- Calculation of the capital charge (CC): $CC = RWA \cdot 8\,\%$

- Risk weighted assets (RWA): $RWA = RW \cdot E^*$

The regulatory capital model separates market scenarios from credit scenarios:

- Market scenarios are aggregated in exposure at default E^*
- Credit scenarios are aggregated in risk weight RW

Current Basel II (CP3) methodology for calculating *E**

"low risk sensitivity, high capital charge"

Basel II Riskhorizon

Time to Maturity (~years)

Remargining Frequency (~days)

$$E^* = \max[0; E - C + Add\text{-}On]$$

OTC-Derivate

$$\text{Add} - \text{On} = Z \quad \ldots \text{ Principle I for Exposure}$$

Collateralised OTC-Derivates

$$\text{Add} - \text{On} = \begin{cases} Z & \ldots \text{ Principle I for Exposure} \\ C \cdot (H_C + H_{C\,FX}) & \ldots \text{ Haircuts for Collateral} \end{cases}$$

Repos without bilateral netting agreement

$$\text{Add} - \text{On} = E \cdot H_E + C \cdot (H_C + H_{C\,FX})$$

Repos with bilateral netting agreement

$$\text{Add} - \text{On} = m \cdot VaR_{99\%/5}$$

m … regulatory multiplier based on backtesting quality

How do we measure exposure under the new Basel Capital Accord ?

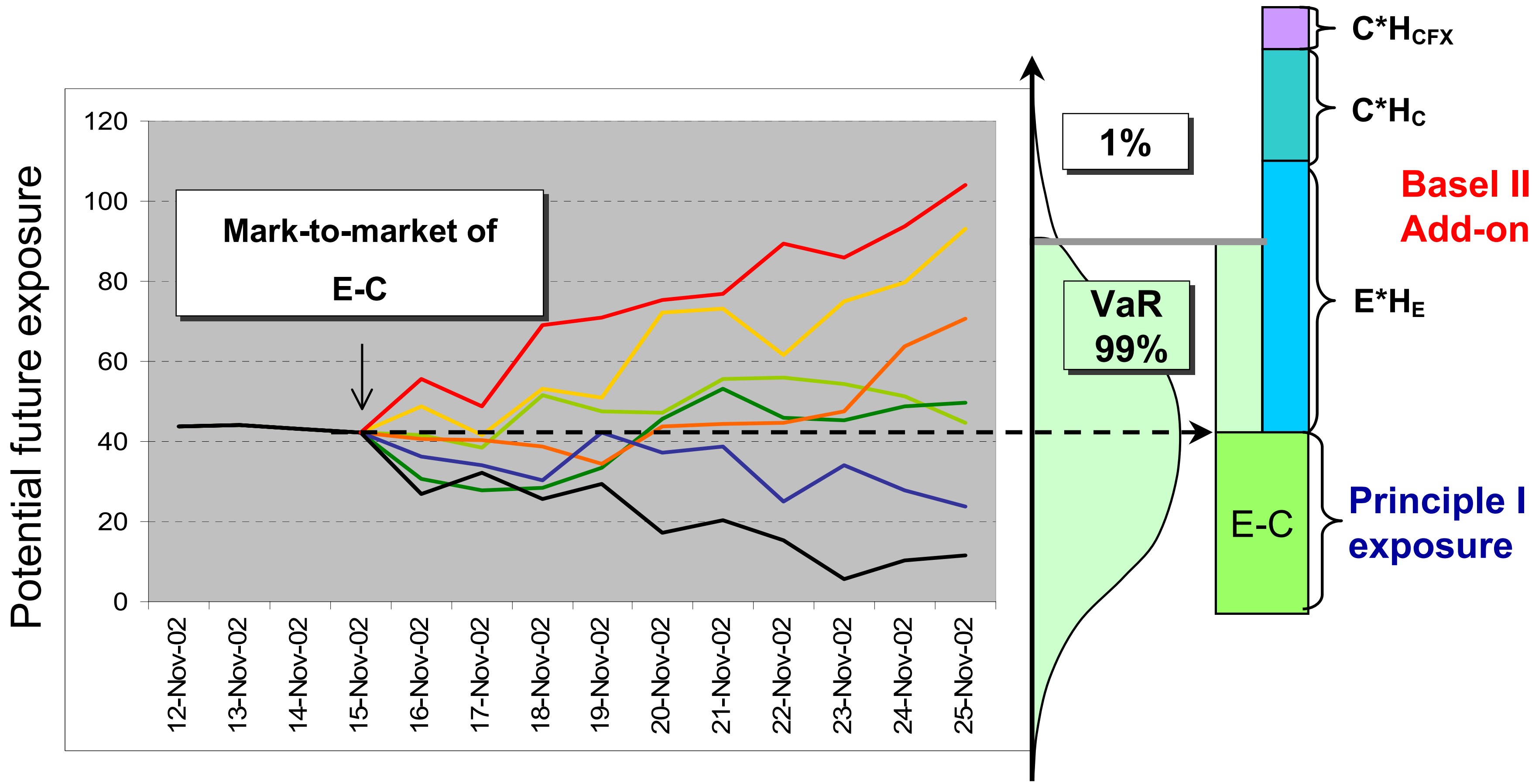
Counterparty risk of capital market transactions

- Exposure value after risk mitigation (E^*) as exposure at default
  - Current exposure (CE) after risk mitigation is an incomplete measure of risk
  - It is important to assess the magnitude of potential future exposures to a counterparty
  - Risk surcharge factors are calibrated at 99%-confidence level

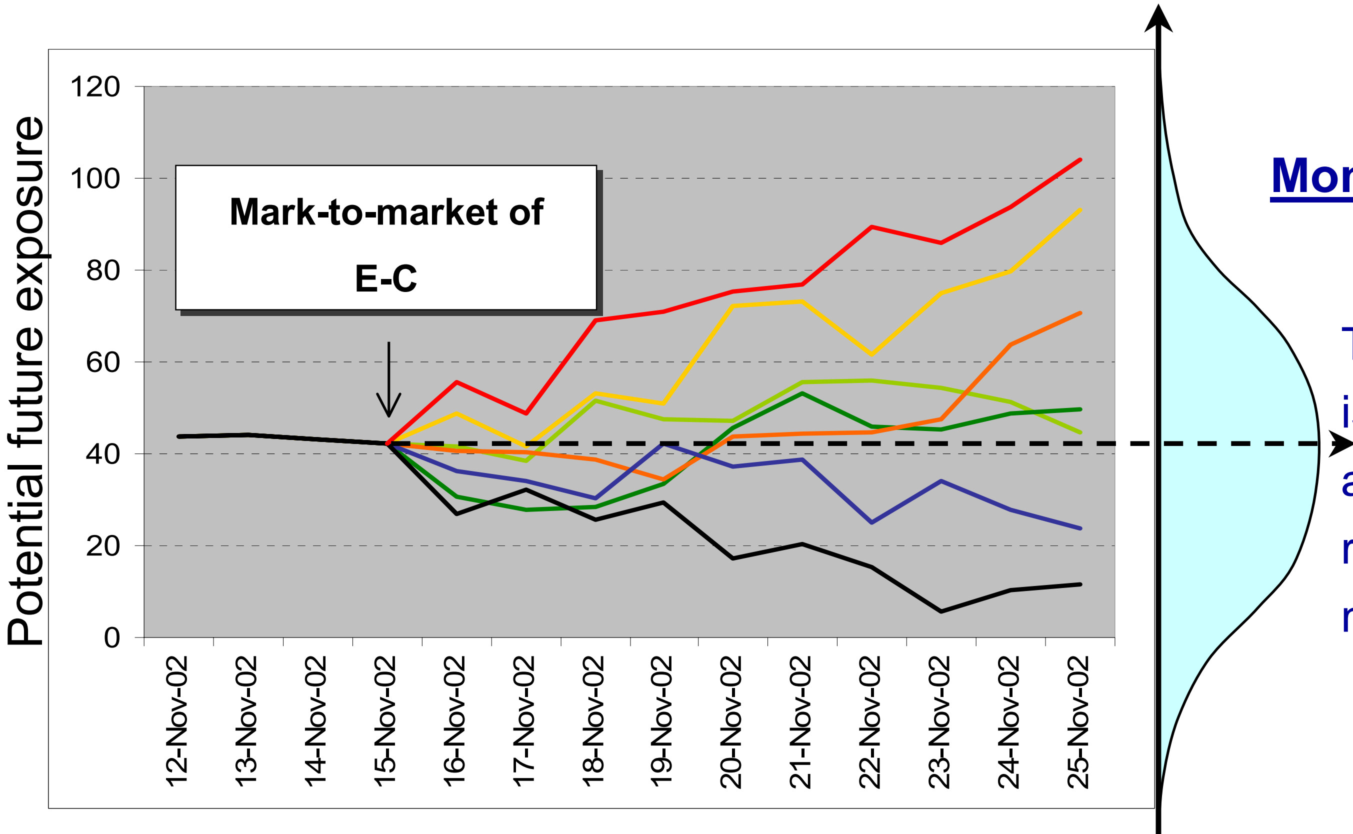
Credit risk of loan transactions

- EAD of commercial loan is calculated as
  - bookvalue of the loan
  - plus contributions from undrawn credit lines that are calibrated as expectation values

Potential future exposure at 99% confidence



Potential future exposure as an expectation value measure (1)



Monte-Carlo sampling of market risk

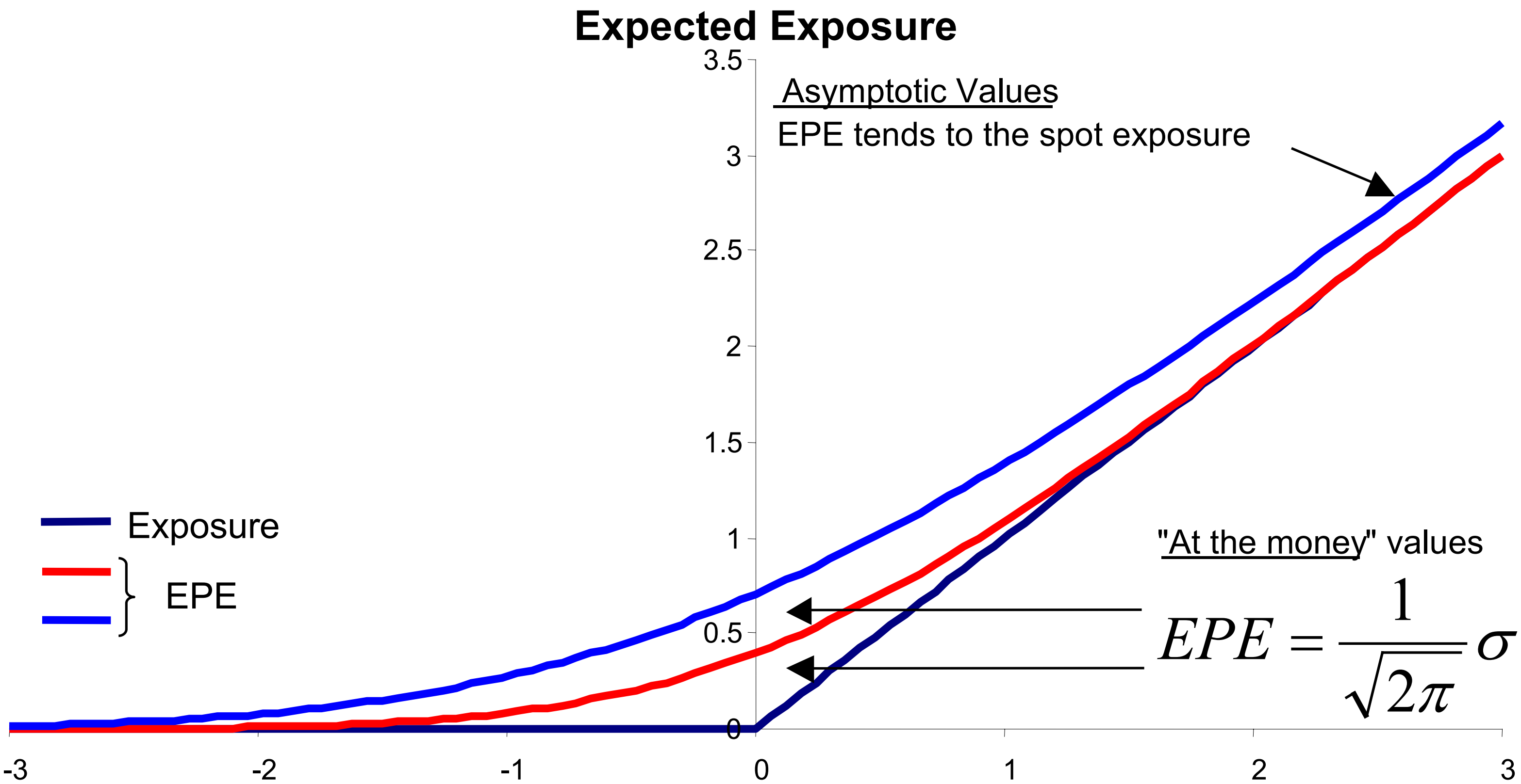
The potential future exposure is calculated by sampling all market scenarios with respect to the normal measure on those paths.

"EPE is similar to call option price"

$$EPE = \mu\left(\max\left[0; \sum_{\text{Exposure} \in \mathrm{N}} E - \sum_{\text{Collateral} \in \mathrm{N}} C \right] \right)$$

Potential future exposure as an expectation value measure (2)

- EPE looks like option on Exposure = Max(0, V(1)).
- EPE actually same as “arithmetic Brownian motion” option valuation (would be Black Scholes if we assumed geometric Brownian motion).



What is the proper loan-equivalent exposure (LEE) amount to capital market transactions ?

- Is it potential future exposure at 99% confidence
    - Haircut approach which neglects correlation across risk factors?
    - or Value at Risk approach which takes correlation effects into account?
- Is it potential future exposure as an expectation value measure
    - Expected positive exposure (EPE) see ISDA (May 2001)?

In order to answer this question level playing field of loans versus capital market has to be ensured !

Level playing field: loans versus capital market products

Counterparty risk of capital market transactions

- The exposure to the counterparty is variable; simulated market scenarios are equal to or exceed a 99% confidence-level exposure with only 1% probability;
- Banks do not have exposures to all counterparties at the same time. Some counterparties owe money to the bank. The bank owes money to others.

Credit risk of loan transactions

- The exposure to an obligor is the expectation value of the loan amount for the entire term of the loan; the estimated loan amount is treated as if it would occur with certainty;
- The total portfolio exposure at any given time is the sum of the exposures on each loan.

Defining exposure on 99%-confidence penalizes capital market transactions because:

- it treats a 1%-probability exposure as if it would occur with certainty;
- ignores the diversification of counterparty exposures across market scenarios as worst case market scenarios are applied to individual counterparty and netting agreements.

Part 2

Based on the expected positive exposure (EPE) approach the proper loan-equivalent exposure (LEE) amount to capital market transactions can be quantified !

What is the LEE for capital market products ?

Asymptotic case

- Infinitely large number of infinitely small counterparty exposures
- Average pair wise correlation between market-driven counterparty exposures is zero
- ISDA response to CP2 (May 2001) shows that:

$$LEE = EPE$$

What is the LEE for capital market products ?

Real-life counterparty portfolios

- Finite number of counterparties and
- Correlated market-driven exposures result in:

$$LEE > EPE$$

$LEE > EPE$: by how much?

- Evan Picoult's (Citigroup) suggestion (see ISDA paper 2003):

$$LEE = \alpha \cdot EPE$$

Calculate the ratio α **= a/b** where:

a = economic capital calculated based on the full simulation of market and credit scenarios, i.e. accounting for the variability of counterparty exposures

b = economic capital calculated based on the simulation of credit scenarios only, assuming that each counterparty exposure is constant and equal to EPE

- Michael Gibson (FED research paper 2002) analyses the understatement of risk (U)

when EPE is used as loan-equivalent amount. ISDA paper demonstrates,

that Gibson's U is related to ISDA's α by

$$U \cong \alpha - 1$$

but his results are smaller as they where obtained for infinite number of counterparties.

Economic capital defines LEE and thus the scaling factor α

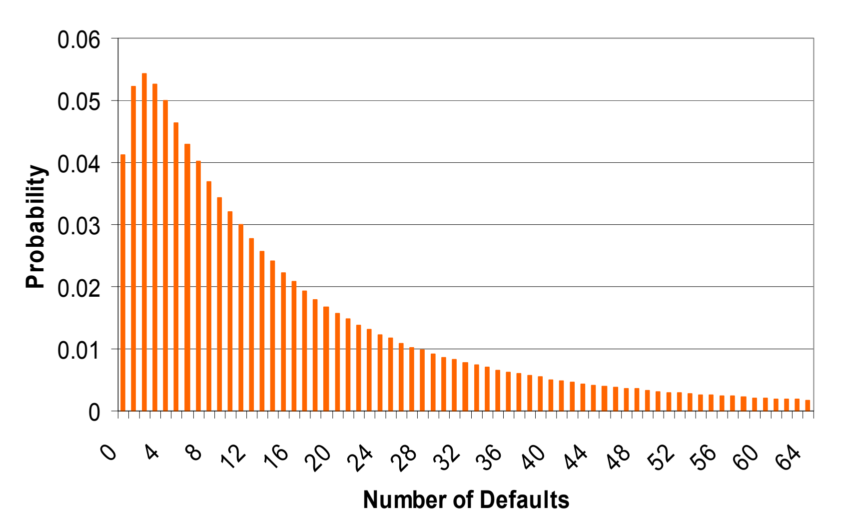
- Counterparty exposures resulting from capital market transactions are variable and market-driven

- Extreme potential credit losses reflect credit and market concentrations in the portfolio of counterparties

The economic capital model should evaluate potential credit losses under the full simulation of joint market and credit scenarios with market scenarios applied coherently to all netting agreements.

Sketch of distribution functions used in economic capital simulation

Default distribution function



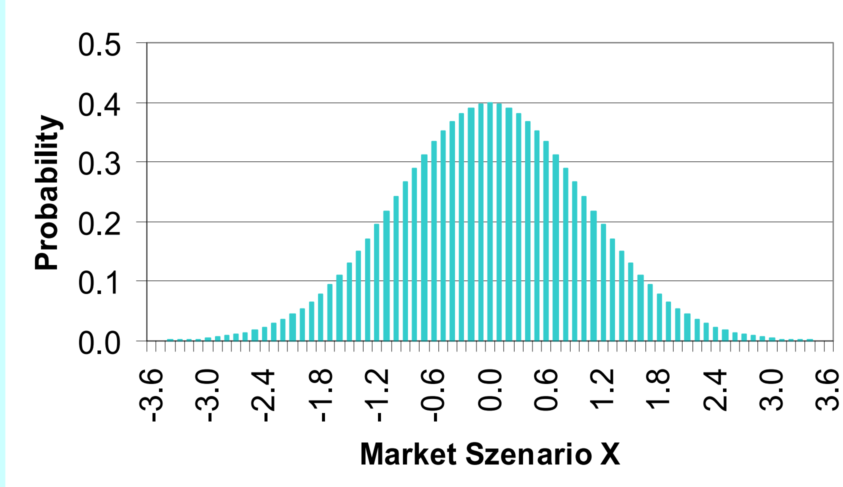
For N=1000; PD=1.85%; R=16.8%

Number of defaulted counterparties 99.9%-confidence level

$$N \cdot PD + q_{\text{Default}} \sqrt{N \cdot PD \cdot (1 - PD)} = 185$$

thus $q_{\text{Default}} = 39.1$

Distribution function for market szenarios



Changes in market risk factors Δ r over the risk horizon from $t_0=0$ to t are modeled from standard normal distributions:

$$\Delta r = \sigma \cdot X \cdot \sqrt{t}$$

where X is the standard normal factor. At 99%-confidence level one has $X_{99\%} = 2.33$.

Outlook on results for α as obtained by ISDA working group

Economic loss calculated on 99.9%-confidence level is mainly determined by the credit scenarios due to fat tails originating from asset correlation R.

Thus, α is found close to 1 with a narrow range from typically 1.05 to 1.25 .

Only in a few extreme portfolio cases, e.g. all counterparties have at the market transactions with $V_A(t=0)=0$, α is larger than 1.5 .

Part 3

Quantitative results from <u>model</u> simulations, industry <u>live portfolios</u>, and <u>analytic</u> methods.

Calculation of expected positive exposure

$V_A(t) = V_A(t=0) + \sum_i \frac{\partial V_A}{\partial r_i} \Delta r_i$	Realized value V of all individual transactions covered by netting agreement A at time t for $i=1, 2 \ldots M$ market risk factors
$E_A(t) = \max\left[V_A(t),\ 0\right]$	Resulting (positive) exposure at time t

$\partial V_A / \partial r_i$ … Sensitivity

$\Delta r_i = \sigma_i \cdot X_i \cdot \sqrt{t}$ … changes in the market risk factor r_i with σ_i

$V_A(t=0) = \sum_{\text{Exposure} \in \text{CA}} (E) - \sum_{\text{Collateral} \in \text{CA}} (C)$ … current mark-to-market of transaction subject to netting agreement A

Monte-Carlo sampling of market risk

The expected positive exposure is the average of $E_A(t) = \max\left[V_A(t), 0\right]$ over all paths with respect to the normal measure X on those paths.

Example calculation of expected positive exposure (see ISDA May 2001)

- Normalize sensitivity $\frac{\partial V_A}{\partial r} = 1$
- At market transaction $V_A(t=0) = 0$

$$E_A(t) = \max\left[\sigma \cdot x \cdot \sqrt{t}, 0\right]$$

- Risk horizon from $t_0=0$ to T, thus the expected exposure at time t is

$$EPE = \mu\left(E_A(t)\right) = \frac{1}{\sqrt{2\pi}} \int_{x=0}^{\infty} \sigma \cdot x \cdot \sqrt{t} \cdot e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \sigma \cdot \sqrt{t}$$

In an at market portfolio EPE contains similar information to counterparty level VaR

$$E^* = \max\left[V_A(t=0) + \sigma \cdot x \cdot \sqrt{t}, 0\right]\Big|_{\substack{V_A(t_0)=0 \\ 99\%-\text{confidence}}} = \sigma \cdot 2.33 \cdot \sqrt{t}$$

Modeling of credit scenarios

$P(X_C) = \Phi\left[\sqrt{1-R}^{\,-1} \cdot \Phi^{-1}(PD) + \sqrt{R/1-R} \cdot X_C\right]$	Vasicek dependence of default probability
$N_{defaulted} = \mathrm{B}\left[N; P(X_C), y\right]$	Number of defaulted counterparties

R … Correlation

X_C … standard normal factor

y … uniformly distributed random number drawn from [0,1]

B … cumulative binomial distribution function (Excel worksheet function CRITBINOM)

N … number of counterparties in the selected PD-class

Loss calculation based on the simulation of market and credit scenarios

- In each default scenario (out of ~ 200.000) , the number of defaulted counterparties $N_{defaulted}$ is randomly selected;
- A market scenario X_M is randomly selected out of ~ 2.000;
- Credit loss of individual transaction A is given by the exposures amount in market scenario X_M over the time horizon t

$$E_A\left(X(M),t\right) = \max\left[V_A(t=0) + \sum_i \frac{\partial V_A}{\partial r_i}\sigma_i \cdot X_i(M)\cdot\sqrt{t},\, 0\right]$$

- The portfolio default loss is the sum of the exposure to all transactions A to defaulted counterparties

$$\sum_{\text{All defaulted transactions } A} E_A\left(X(M),t\right)$$

- Economic capital is the total portfolio loss on 99.9% - confidence level based on ~ 200.000 default scenarios

What are the portfolio characteristics that determine the value of α ?

- Number of counterparties in the bank’s portfolio
- Correlations among counterparty exposures
- Correlations among counterparty credit qualities
- Level of current exposures
- Granularity of the portfolio of exposures
- Probability of default
- Confidence level

Model Portfolio

Dresdner Bank and Goldman Sachs Monte-Carlo simulation

- Random normalized (*) sensitivity $S_i(A)$
- At market transaction $V_A(t=0)=0$

$$E_A(t)=\max\left[\sum_{i=1}^{M} S_i(A)\cdot\sigma_i\cdot x_i\cdot\sqrt{t},\,0\right]$$

(*) $\sqrt{\sum_{i=1}^{M} S_i(A)^2}=1$

Typical large-dealer portfolio

- 200 effective counterparties N
- 10 independent market risk factors M
- PD = 0.0020 (approx. BBB) over 1 year
- Pair wise correlation among default drivers R = 0.22
- Maximum Potential Exposure (95%) / Current Exposure = 1.3 over 1 year
- Confidence level = 99.9%

$$\alpha = 1.09$$

Sensitivity of α with respect to various parameters from model simulation

CE	MPE(95%)/CE	α	
0	-	1.32	
1	1.43	1.15	
1.17	**1.31**	**1.09**	**(base case)**
2	1.08	1.03	

R	α	
12%	1.31	
18%	1.22	
22%	**1.09**	**(base case)**

M	α	
5	1.10	
10	**1.09**	**(base case)**
20	1.07	

N	α	
100	1.17	
200	**1.09**	**(base case)**
500	1.06	

PD	α	
0.11%	1.07	
0.20%	**1.09**	**(base case)**
0.60%	1.06	

Industry live Derivatives Portfolios

International Swaps and Derivatives Association Market Survey

	Measured Alpha from Derivatives Portfolios
Model Base Case	1.09
Firm 1	1.08
Firm 2	1.1
Firm 3	1.09
Firm 4	1.07
Firm 5	1.07

Analytic Results

Tom Wilde Credit Suisse First Boston based on granularity adjustment method

- Excellent Agreement between simulated values for α and analytic results have been obtained over wide range of parameters;
- Analytic α for the model base case is $\alpha = 1.08$

Part 4

Summary

Summary of industry proposal to regulators

- Calculation of the capital charge (CC): $CC = RWA \cdot 8\,\%$
- Risk weighted assets (RWA): $RWA = RW \cdot E^*$
- Measure counterparty exposure as with α fixed by regulators:

$$E^* = \alpha \cdot EPE$$

- Financial industry agrees (based on simulations) that α is close to 1 with a narrow range from typically 1.05 to 1.25 (see ISDA paper 2003);
- EPE-approach ensures level playing field of loans versus capital market products;
- EPE calculation is based on coherent market scenarios $\Rightarrow$ diversification across counterparty and netting agreement is accounted for;
- EPE-approach can be applied to all capital market products and netting agreements and does allow for cross product netting $\Rightarrow$ reduces systemic risk.

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