FX Quantitative Research

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Stochastic volatility models: A Finite Difference Approach

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Exotic Option Pricing Example: Onetouch

Payout at Maturity: $1_{\{\text{for all } t<T: S\_t>L\}}$

the seller of the one touch pays 1,000,000$ at maturity if USD/JPY spot 132.40 ever trades above a given level 164 before the maturity 18.03.2003 of the one touch

TV 2.6%=26,000$ paid at 5%=50,000$

market was at 4,5% - 6,5%

Exotic Option Pricing Example: Onetouch

Pricerequest: Vega-hedge is included !

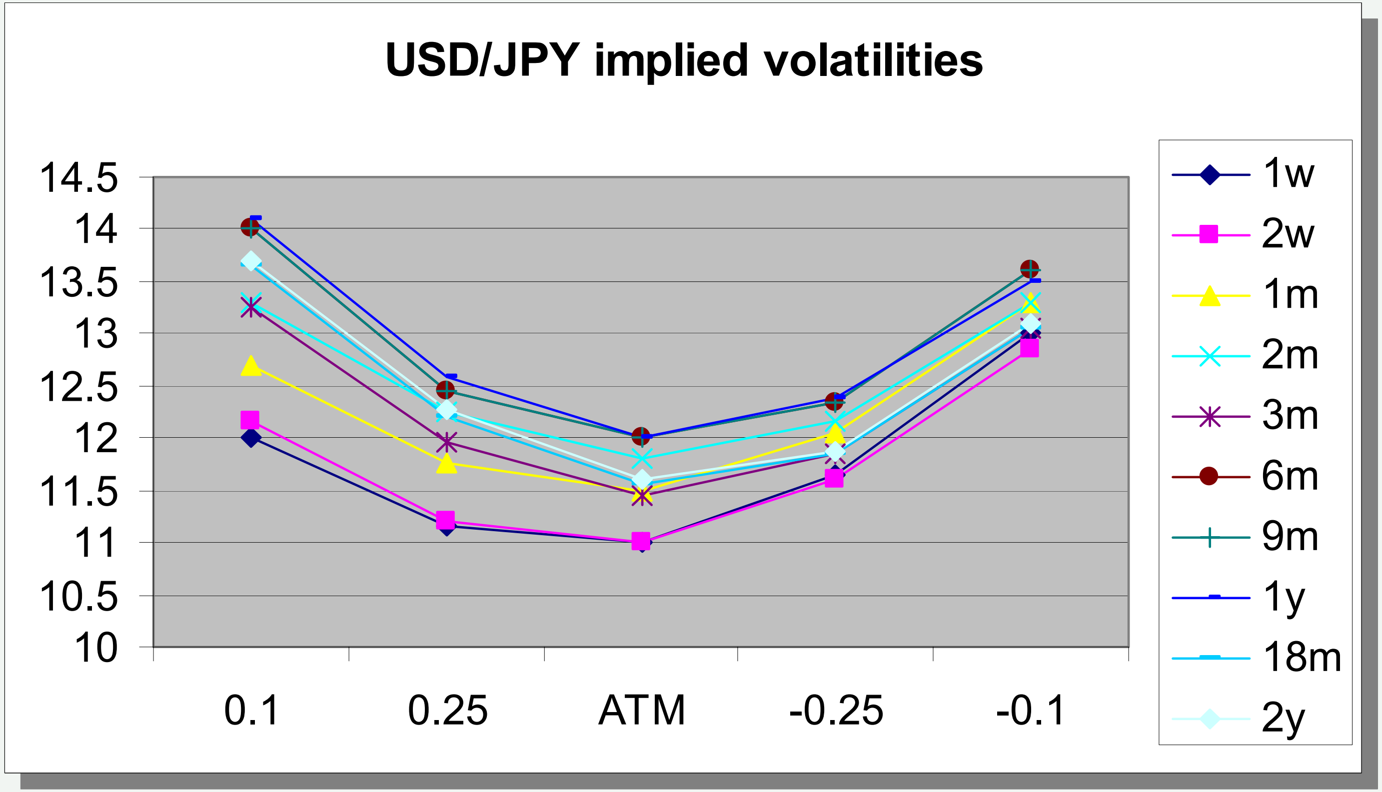
1Y (18 Mar) $/YEN 164 OT PAYS $1MIO AT MAT SPT 132.40
VOL 10.50, TV 2.6%, DELTA +63, NY CUT, PLS VH

4.5-6.5 VH

The Market already takes into account that prices are not consistent with the BSM model

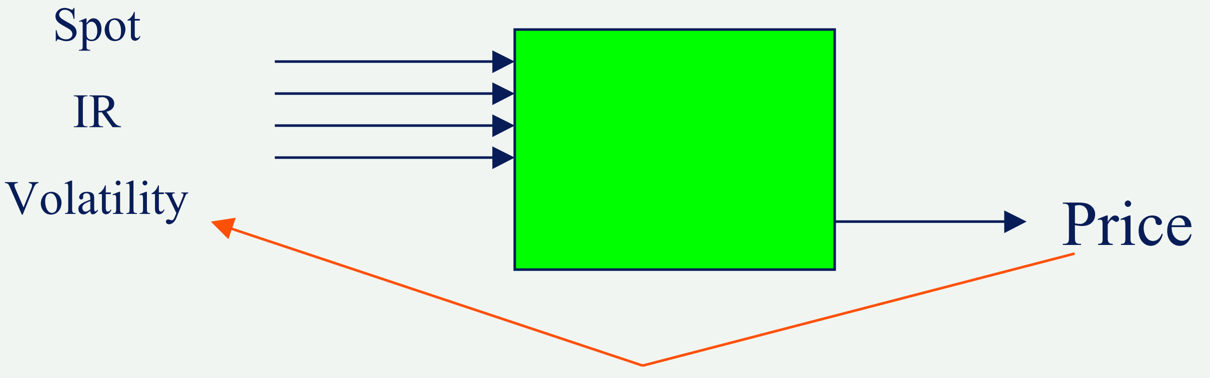
- volatility is the second most important risk factor with spot being number one
- market makers take into account that volatility is not constant (vega-hedge for exotics)
- Assumption: The smile for vanilla options includes all information necessary to price exotics

The Volatility Smile for Plain Vanilla Options



The Volatility Smile for Plain Vanilla Options

- All reasons for a different price are discounted into one variable: Volatility
- The smile expresses the relative price of Out-of-the-money plain vanilla options to the at-the-money options in terms of implied volatility



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Heston's Stochastic Volatility Model

$$dS_t = S_t \left[\mu \, dt + \sqrt{v(t)} dW_t^{(1)} \right]$$

$$dv_t = \kappa(\theta - v_t) \, dt + \xi \sqrt{v(t)} dW_t^{(2)}$$

$$\mathrm{Cov} \left[dW_t^{(1)}, dW_t^{(2)} \right] = \rho \, dt$$

- Spot: diffusion process with
  - drift: interest rate differential
  - stochastic instantaneous volatility
- Variance: diffusion process with
  - mean reversion
  - long run variance
  - volatility of variance
- Correlation between variance and spot

Solution for Plain Vanilla Options

$$\mathrm{HestonVanilla}(\kappa, \theta, \xi, \rho, \lambda, r_d, r_f, v_t, S_t, K, \tau, \phi)$$
$$= \phi\left[e^{-r_f \tau} S_t P_+(\phi) - K e^{-r_d \tau} P_-(\phi)\right]$$

$$P_+(\phi) = \tfrac{1-\phi}{2} + \phi P_1(\ln S_t, v_t, \tau, \ln K)$$
$$P_-(\phi) = \tfrac{1-\phi}{2} + \phi P_2(\ln S_t, v_t, \tau, \ln K)$$

$$P_j(x, v, \tau, y) = \tfrac{1}{2} + \tfrac{1}{\pi} \int_0^\infty \Re\left[\frac{e^{-i\varphi y} f_j(x, v, \tau, \varphi)}{i\varphi}\right] d\varphi$$

$$f_j(x, v, t, \varphi) = e^{C_j(\tau,\varphi) + D_j(\tau,\varphi) v + i\varphi x}$$

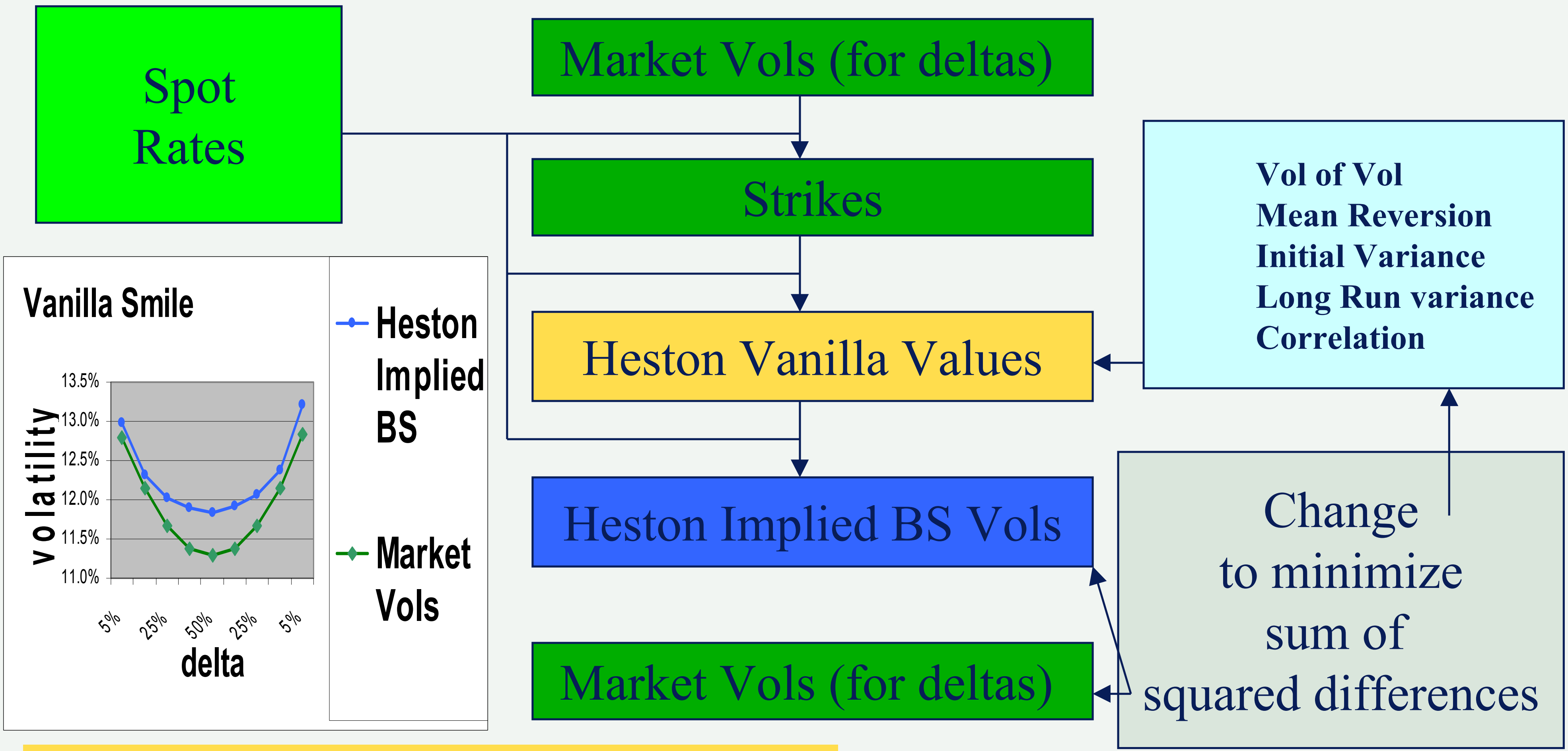
- ‘Closed’ form solution

Calibration

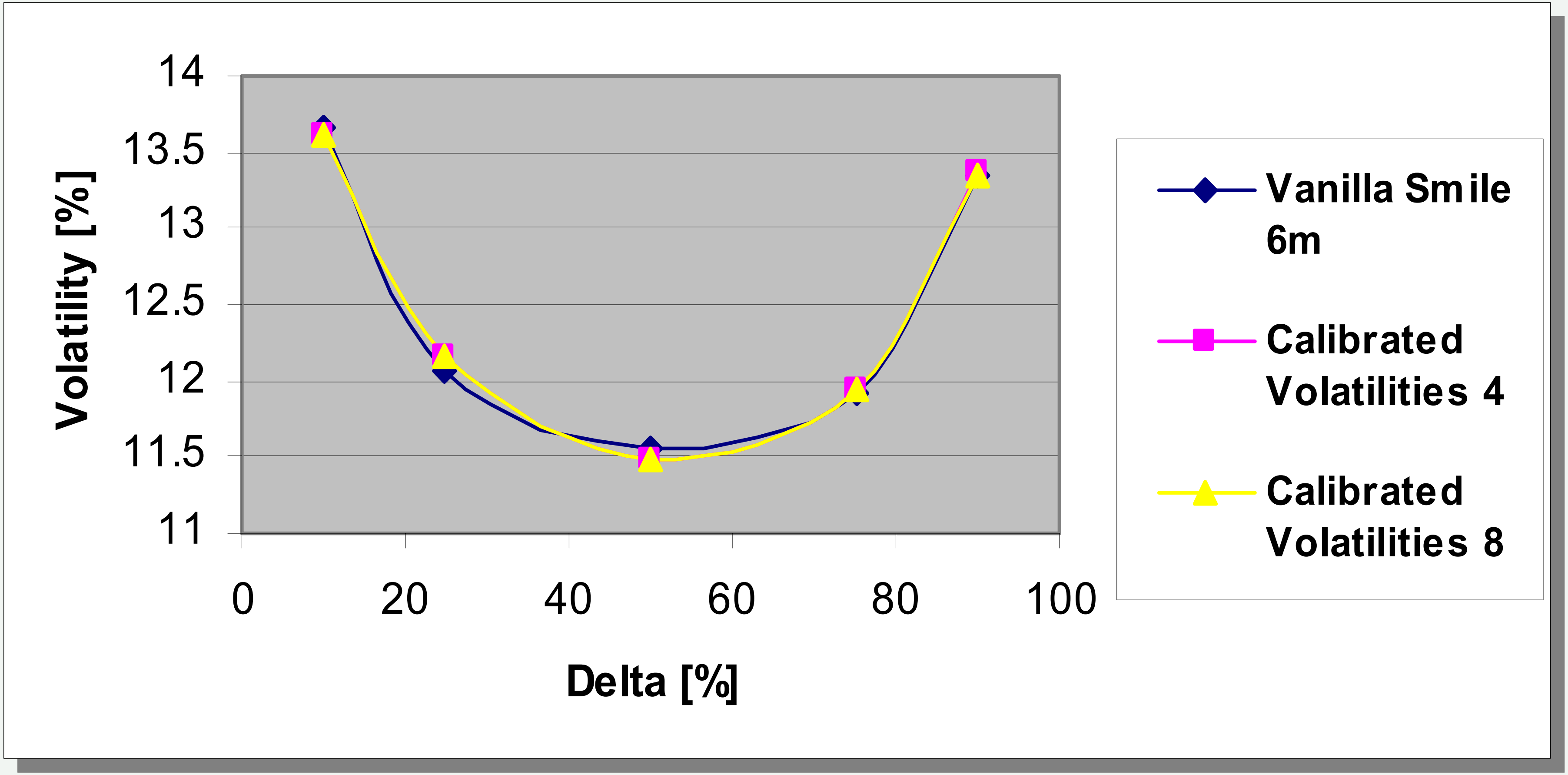
- Calibration means: finding the parameters for the model.
- Fitting to market prices: Implied volatilities of ‚liquid' plain vanilla options
- Method of fitting: Mean Squared Minimization of market implied vols and model implied vols

$$\min_{\kappa,\xi,\theta,v_0,\rho} \sum_{i=1}^{n} ||\sigma_i - \sigma(P_i)||^2$$

Calibration Engine



Calibration on 6m USD/JPY



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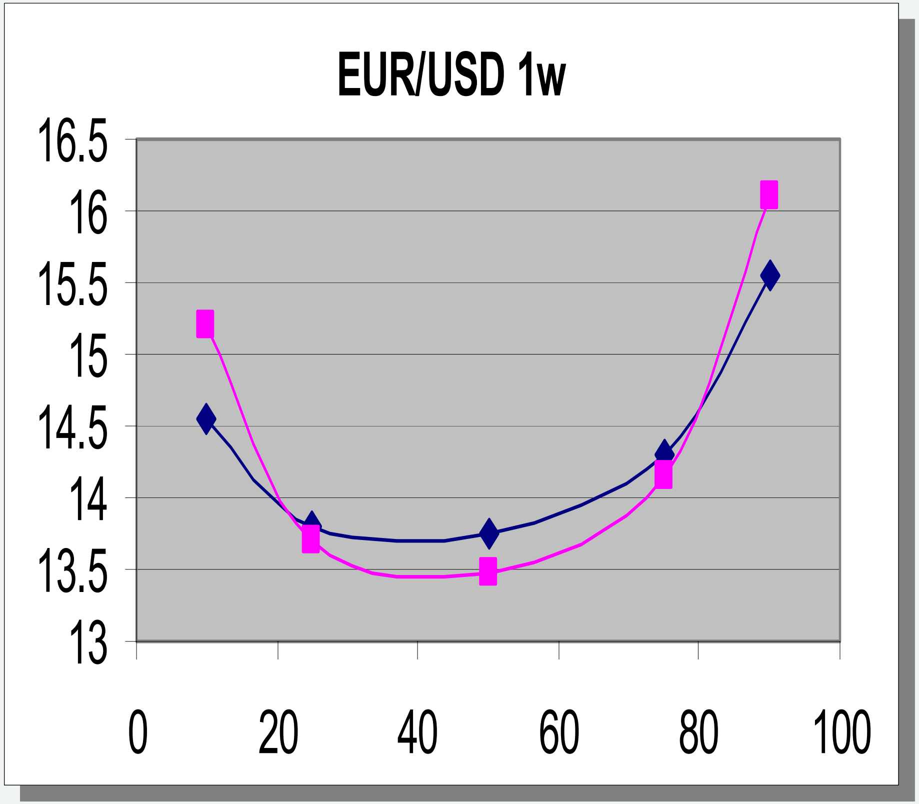
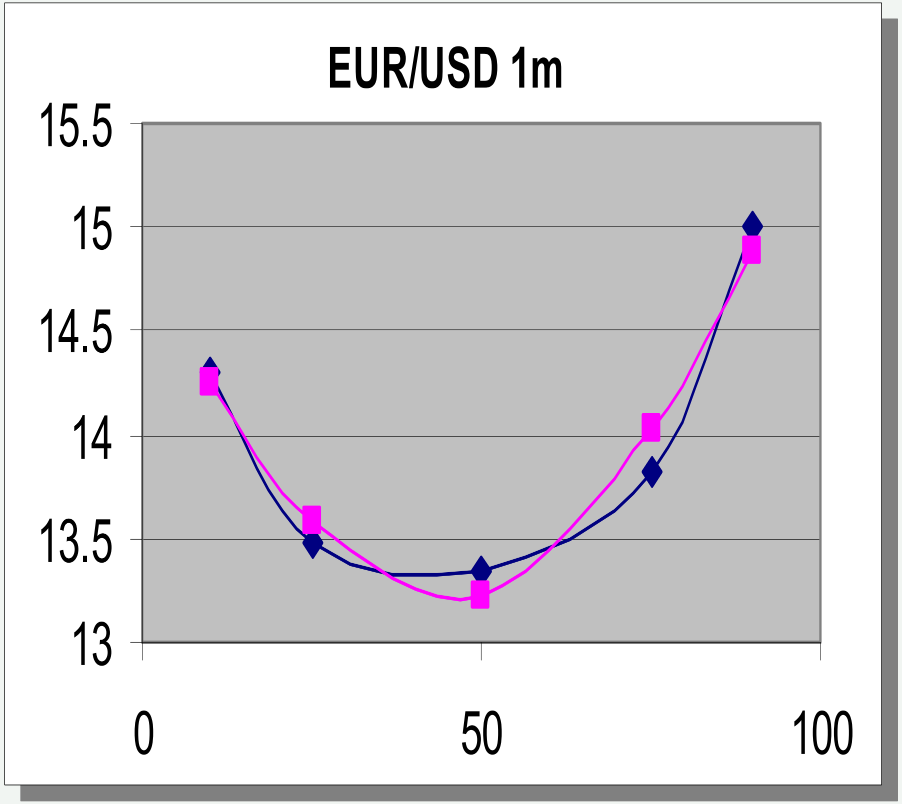
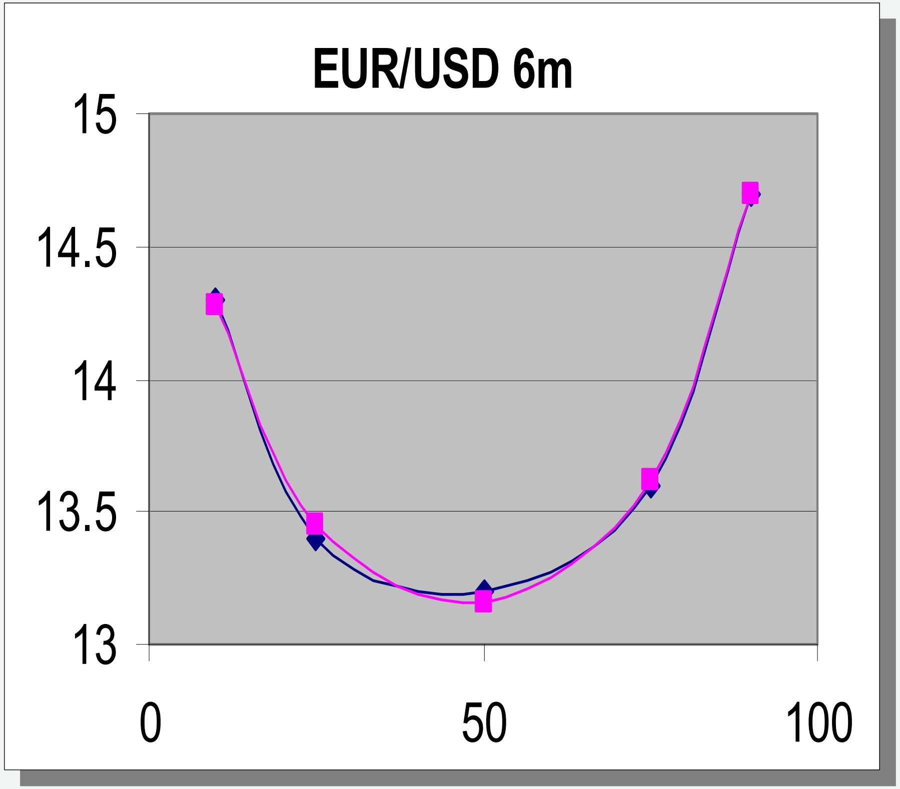
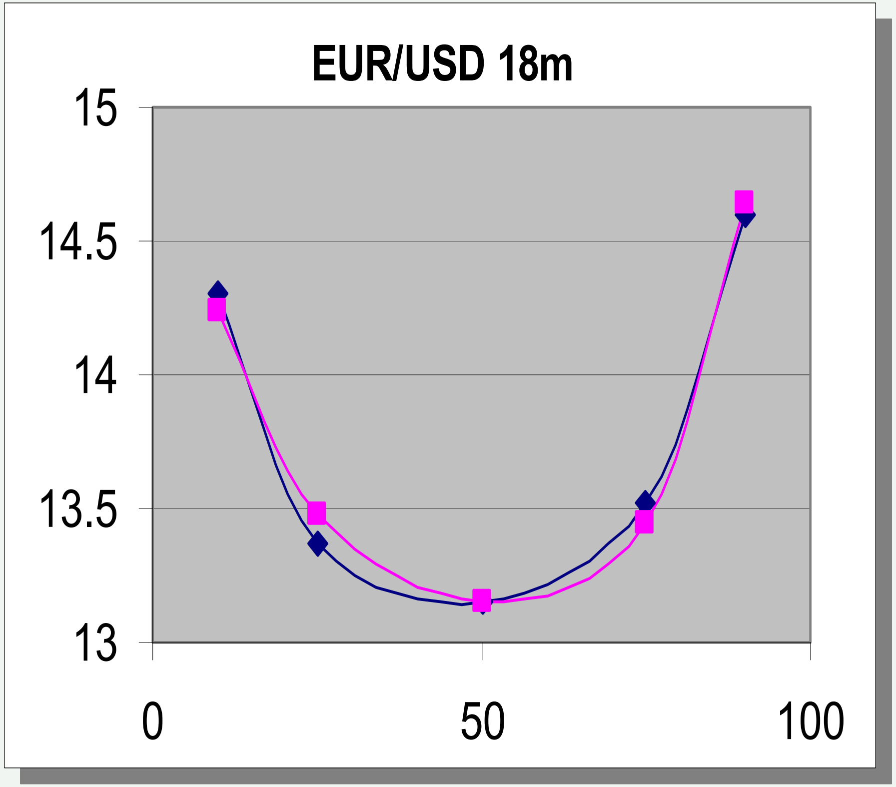
EUR/USD- Calibration on short maturities





EUR/USD- Calibration on longer maturities

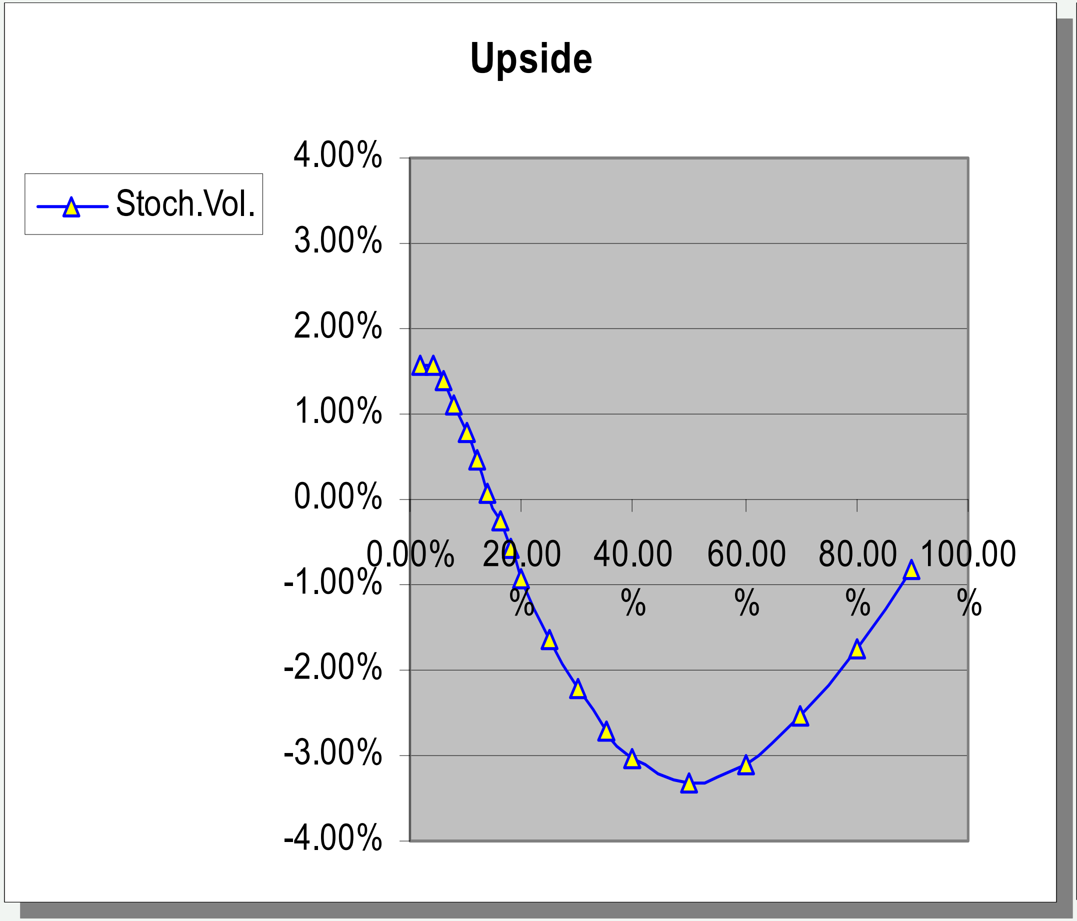


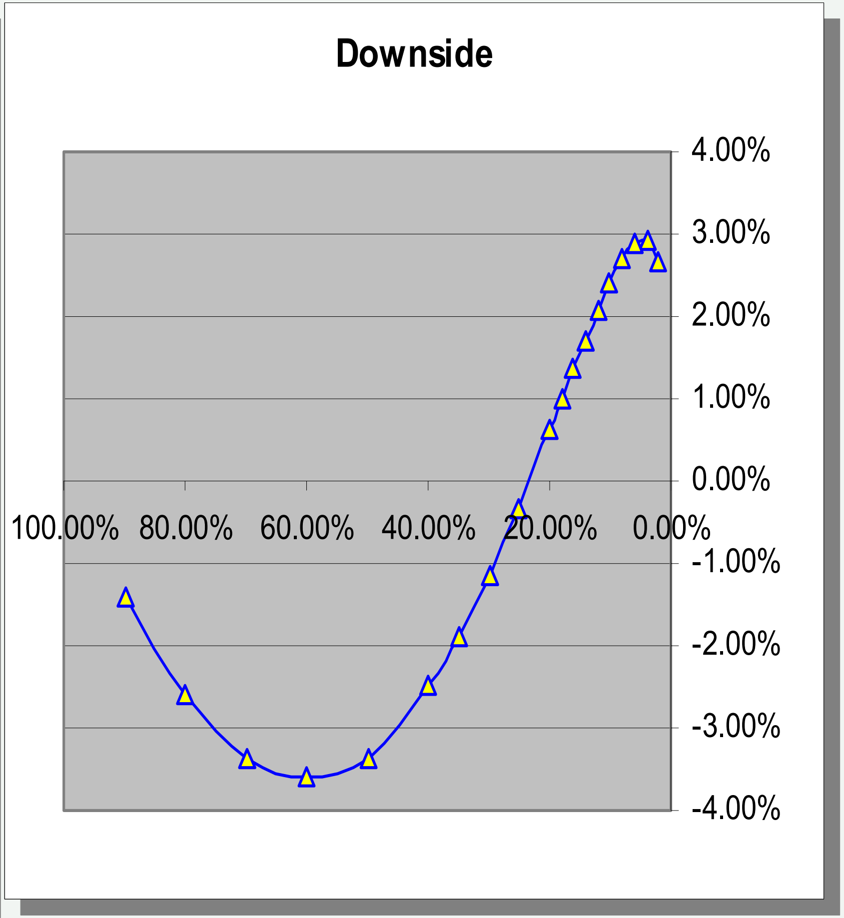


Pricing of exotics

- Closed form solution not available
- Monte Carlo
- Solve the PDE - finite difference
- Trees - trinomial

Results for EUR/USD One Touch 1m





Standard Finite Difference Method

- works
- rather slow