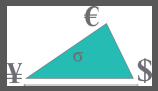

Vedic Mathematics

Uwe Wystup

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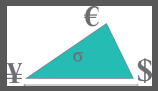
February 2011

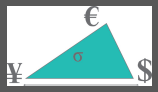


1. Introduction

veda (Sanskrit) means: knowledge

Veda	Upaveda
Rigveda	Ayurveda
Samaveda	Gandharvaveda
Yajurveda	Dhanurveda
Atharvaveda	Sthapatyaveda

Table 1: Vedas and Upavedas (supplementary vedas)



1.1. The 16 *Sutras*

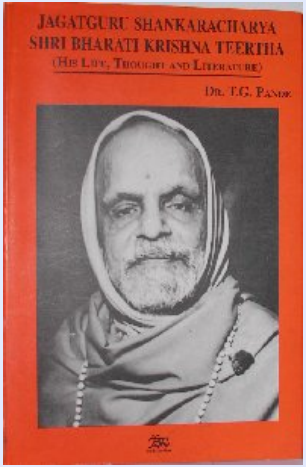
are part of a *Parisista* (Appendix) of the *Atharvaveda*

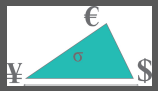
1. By one more than the one before
2. All from 9 and the last from 10
3. Vertically and crosswise
4. Transpose and apply
5. If the Samuccaya is the same it is zero
6. If one is in ratio the other is zero
7. By addition and by subtraction
8. By the completion or non-completion
9. Differential calculus
10. By the deficiency
11. Specific and general
12. The remainders by the last digit
13. The ultimate and twice the penultimate
14. By one less than the one before
15. The product of the sum
16. All the multipliers

1.2. Jagadguru Swami Sri Bharati Krsna Tirthaji Maharaja

Explained the sutras in his books (e.g. [2]).

Jagadguru Swami Sri Bharati Krsna Tirthaji Maharaja (March, 1884 - February 2, 1960) was the Jagadguru (literally, teacher of the world; assigned to heads of Hindu mathas) of the Govardhana matha of Puri during 1925-1960. He was one of the most significant spiritual figures in Hinduism during the 20th century. He is particularly known for his work on Vedic mathematics.





2. Multiplication

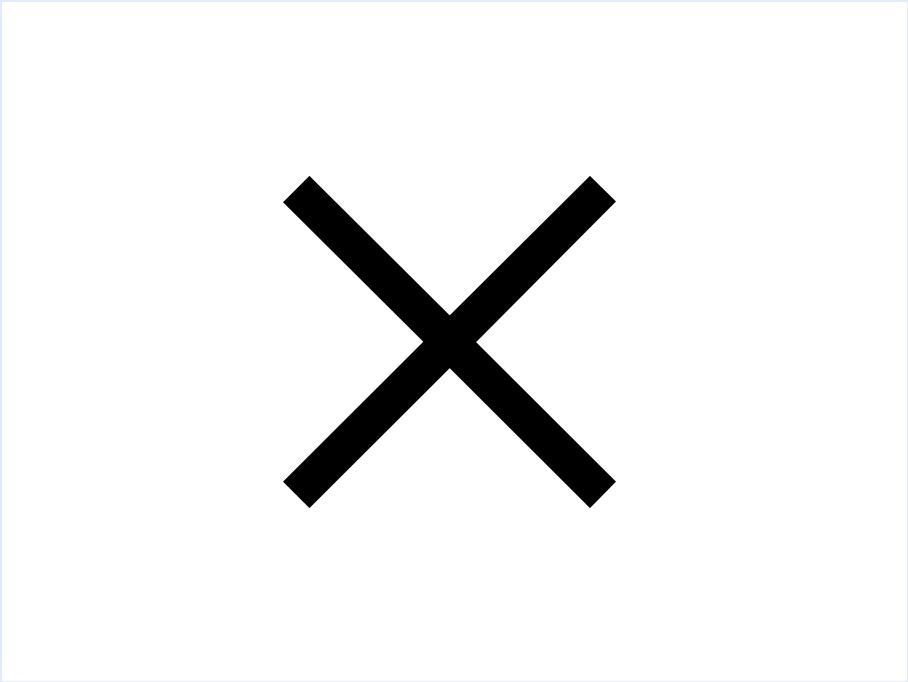
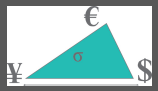


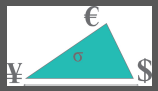
Figure 1: Vertically and Crosswise



2.1. Example with working base 10

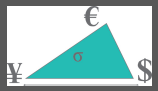
$$\begin{array}{rcl} 9 & - & 1 \\ & \times & \\ 7 & - & 3 \\ \hline 6 & / & 3 \end{array}$$

$$= 63$$



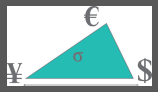
2.1. Example with working base 10

$$\begin{array}{rcl} 9 - 1 & & 7 - 3 \\ \times & & \times \\ 7 - 3 & & 6 - 4 \\ \hline 6 \;/\; 3 & & 3 \;/_1\; 2 \\ = 63 & & = 42 \end{array}$$



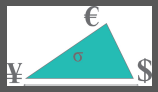
2.1. Example with working base 10

$$\begin{array}{rcr} 9 - 1 & 7 - 3 & 13 + 3 \\ \times & \times & \times \\ 7 - 3 & 6 - 4 & 12 + 2 \\ \hline 6 \;/\; 3 & 3 \;/_1\; 2 & 15 \;/\; 6 \\ = 63 & = 42 & = 156 \end{array}$$



2.1. Example with working base 10

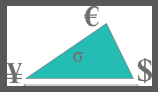
$$\begin{array}{rcr} 9 & - & 1 \\ & \times & \\ 7 & - & 3 \\ \hline 6 & / & 3 \\ & = & 63 \end{array} \qquad \begin{array}{rcr} 7 & - & 3 \\ & \times & \\ 6 & - & 4 \\ \hline 3 & /_1 & 2 \\ & = & 42 \end{array} \qquad \begin{array}{rcr} 13 & + & 3 \\ & \times & \\ 12 & + & 2 \\ \hline 15 & / & 6 \\ & = & 156 \end{array} \qquad \begin{array}{rcr} 12 & + & 2 \\ & \times & \\ 8 & - & 2 \\ \hline 10 & / & \bar{4} \\ & = & 96 \end{array}$$



2.1. Example with working base 10

9	-	1		7	-	3		13	+	3		12	+	2	
	×				×				×				×		
7	-	3		6	-	4		12	+	2		8	-	2	
6	/	3		3	$/_1$	2		15	/	6		10	/	$\bar{4}$	
	=	63			=	42			=	156			=	96	

Reason: $(x+a)(x+b) = x(x+a+b)+ab$

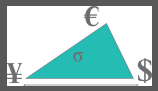


2.1. Example with working base 10

9	-	1		7	-	3		13	+	3		12	+	2	
	×				×				×				×		
7	-	3		6	-	4		12	+	2		8	-	2	
6	/	3		3	$/_1$	2		15	/	6		10	/	$\bar{4}$	
	=	63			=	42			=	156			=	96	

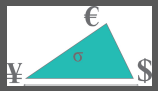
Reason: $(x+a)(x+b) = x(x+a+b)+ab$

Origin of the $\times$-sign comes from this method



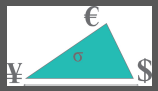
2.2. Example with working base 100

$$\begin{array}{rcr} 91 & - & 9 \\ & \times & \\ 96 & - & 4 \\ \hline 87 & / & 36 \\ & = & 8736 \end{array}$$



2.2. Example with working base 100

91	-	9	111	+	11
	×			×	
96	-	4	109	+	9
87	/	36	120	/	99
	=	8736		=	12099

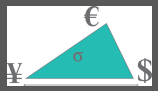


2.2. Example with working base 100

91	-	9	111	+	11	108	+	8
	×			×			×	
96	-	4	109	+	9	97	-	3
87	/	36	120	/	99	105	/	$\bar{2}\bar{4}$
	=	8736		=	12099		=	10476

2.3. Other working bases (division case)

$$
\begin{array}{|ccc}
100/2{=}50 & & \\
\hline
49 & - & 1 \\
 & \times & \\
49 & - & 1 \\
\hline
2)48 & / & 01 \\
24 & / & 01 \\
 & = & 2401
\end{array}
$$



2.3. Other working bases (division case)

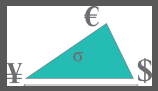
$100/2{=}50$			$100/2{=}50$		
49	-	1	54	+	4
	$\times$			$\times$	
49	-	1	46	-	4
2)48	/	01	2)50	/	$\bar{1}\bar{6}$
24	/	01	25	/	$\bar{1}\bar{6}$
	$=$	2401		$=$	2484

2.4. Other working bases (multiplication case)

$$
\begin{array}{rcl}
10\times\ 2{=}20 & & \\
\hline
19 & - & 1 \\
 & \times & \\
19 & - & 1 \\
\hline
\times\ 2)18 & / & 1 \\
36 & / & 1 \\
 & = & 361
\end{array}
$$

2.4. Other working bases (multiplication case)

$10\times\ 2=20$			$10\times\ 6=60$		
19	-	1	62	+	2
	$\times$			$\times$	
19	-	1	48	-	12
$\times\ 2)18$	/	1	$\times\ 6)50$	$/_{\bar{2}}$	$\bar{4}$
36	/	1	300	$/_{\bar{2}}$	$\bar{4}$
	$=$	361		$=$	2976



2.5. Exercises

Multiply the following

a 94×94

b 97×89

c 87×99

d 87×98

e 87×95

f 95×95

g 79×96

h 98×96

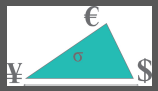
i 92×99

j 88×88

k 97×56

l 97×63

m 92×196



Multiply the following mentally

a 667×998

b 768×997

c 989×998

d 885×997

e 883×998

f 8×6

g 891×989

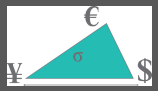
h 8888×9996

i 6999×9997

j 90909×99994

k 78989×99997

l 9876×9989



Multiply the following mentally

a 133×103

b 107×108

c 171×101

d 102×104

e 132×102

f 14×12

g 18×13

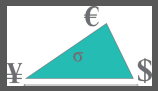
h 1222×1003

i 1051×1007

j 15111×10003

k 125×105

l 10607×10008

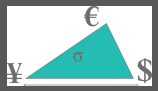


Multiply the following mentally

a 667×998

b 78989×99997

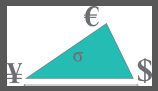
c 1222×1003



3. Squares

Using the sutra *Ekadhikena Purvena* ("by one more than the previous one") we get

$$\begin{aligned} 15^2 &= 1 \times 2/25 = 2/25 = 225 \\ 25^2 &= 2 \times 3/25 = 6/25 = 625 \\ 35^2 &= 3 \times 4/25 = 12/25 = 1225 \\ &\vdots \\ 115^2 &= 11 \times 12/25 = 132/25 = 13225 \end{aligned}$$



3.1. Exercises

Multiply the following mentally

a 65^2

b 85^2

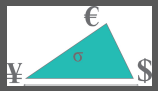
c 0.5^2

d 7.5^2

e 0.0225^2

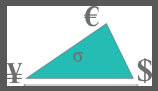
f 1050^2

g 175^2



4. Division

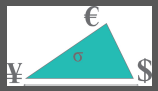
- Find the *exact* decimal representation of $\frac{1}{19}$.



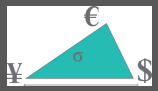
4. Division

- Find the *exact* decimal representation of $\frac{1}{19}$.
- Using the "Ekadhika Purva" Sutra it is easy:

.	0	5	2	6	3	1	5	7	8
1		1			1	1	1	1	
/	9	4	7	3	6	8	4	2	1
	1		1	1					

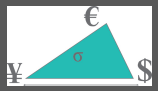


- Start with 1 and then work from right to left multiplying by 2.



- Start with 1 and then work from right to left multiplying by 2.

	.	0	5	2	6	3	1	5	7	8
	1		1			1	1	1	1	
/	9	4	7	3	6	8	4	2	1	
		1		1	1					

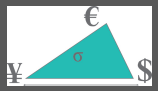


- A further shortcut is the insight that

$$\begin{array}{rcccccccccc} & . & 0 & 5 & 2 & 6 & 3 & 1 & 5 & 7 & 8 \\ + & 9 & 4 & 7 & 3 & 6 & 8 & 4 & 2 & 1 & \\ \hline = & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & \end{array}$$

- The same works for all periodic decimals, e.g. $\frac{1}{7}$

$$\begin{array}{rcccc} & . & 1 & 4 & 2 \\ + & 8 & 5 & 7 & \\ \hline = & 9 & 9 & 9 & \end{array}$$

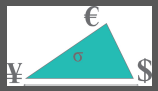


4.1. Exercises

Compute the *exact* decimal number of

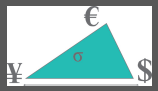
a $\frac{1}{29}$

b $\frac{1}{49}$



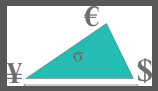
5. Divisibility

- Use Ekadhika as an osculator.



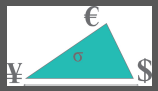
5. Divisibility

- Use Ekadhika as an osculator.
- For 9, 19, 29, 39 etc. the Ekadhikas are 1, 2, 3, 4, etc.



5. Divisibility

- Use Ekadhika as an osculator.
- For 9, 19, 29, 39 etc. the Ekadhikas are 1, 2, 3, 4, etc.
- For 3, 13, 23, 33 etc. multiply them by 3 and you get 1, 4, 7, 10, etc. as the Ekadhikas.



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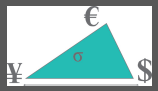
- For 3, 13, 23, 33 etc. multiply them by 3 and you get 1, 4, 7, 10, etc. as the Ekadhikas.
- For 7, 17, 27, 37 etc. multiply them by 7 and you get 5, 12, 19, 26, etc. as the Ekadhikas.



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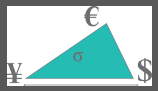
- For 3, 13, 23, 33 etc. multiply them by 3 and you get 1, 4, 7, 10, etc. as the Ekadhikas.
- For 7, 17, 27, 37 etc. multiply them by 7 and you get 5, 12, 19, 26, etc. as the Ekadhikas.
- For 1, 11, 21, 31 etc. multiply them by 9 and you get 1, 10, 19, 28, etc. as the Ekadhikas.



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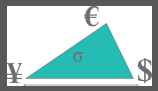
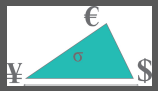
- For 3, 13, 23, 33 etc. multiply them by 3 and you get 1, 4, 7, 10, etc. as the Ekadhikas.
- For 7, 17, 27, 37 etc. multiply them by 7 and you get 5, 12, 19, 26, etc. as the Ekadhikas.
- For 1, 11, 21, 31 etc. multiply them by 9 and you get 1, 10, 19, 28, etc. as the Ekadhikas.
- Now test if 112 is divisible by 7 osculating by 5: $2 \times 5 + 11 = 21$, which is divisible by 7.
- Therefore: yes



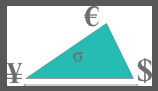
5. Divisibility

- Use Ekadhika as an osculator.
- For 9, 19, 29, 39 etc. the Ekadhikas are 1, 2, 3, 4, etc.
- For 3, 13, 23, 33 etc. multiply them by 3 and you get 1, 4, 7, 10, etc. as the Ekadhikas.
- For 7, 17, 27, 37 etc. multiply them by 7 and you get 5, 12, 19, 26, etc. as the Ekadhikas.
- For 1, 11, 21, 31 etc. multiply them by 9 and you get 1, 10, 19, 28, etc. as the Ekadhikas.
- Now test if 112 is divisible by 7 osculating by 5: $2 \times 5 + 11 = 21$, which is divisible by 7.
- Therefore: yes

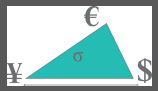


Is 2774 divisible by 19? Osculate by 2:

$$\begin{array}{r} 2\ 7\ 7\ 4 \\ +\ 8 \\ \hline 2\ 8\ 5 \\ +\ 1\ 0 \\ \hline 3\ 8 \\ +\ 1\ 6 \\ \hline 1\ 9 \end{array}$$



- One more example: Is 5293240096 divisible by 139?
- The Ekadhika (osculator) is 14.



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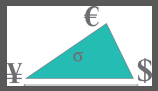
5	2	9	3	2	4	0	0	9	6
139	89	36	131	29	131	19	51	93	



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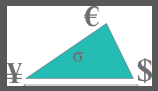
- The Ekadhika (osculator) is 14.

5	2	9	3	2	4	0	0	9	6
139	89	36	131	29	131	19	51	93	

- Answer: yes



5.1. Exercises

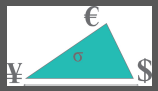
Using the osculation method, check if

a 32896 is divisible by 29

b 93148 is divisible by 29

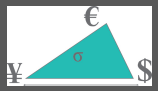
c 4914 is divisible by 39

d 14061 is divisible by 43



6. Square Roots (Vargamula)

- Square numbers only have digit sums $1, 4, 7, 9$



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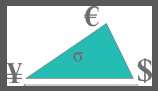
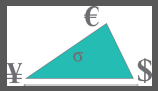
- Square numbers only have digit sums $1, 4, 7, 9$
- and they only end in $1, 4, 5, 6, 9, 0$.



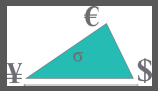
6. Square Roots (Vargamula)

- Square numbers only have digit sums 1, 4, 7, 9
- and they only end in 1, 4, 5, 6, 9, 0.
- If the given number has n digits, then the square root will contain $\frac{n}{2}$ or $\frac{n+1}{2}$ digits.



6. Square Roots (Vargamula)

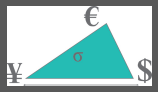
- Square numbers only have digit sums $1, 4, 7, 9$
- and they only end in $1, 4, 5, 6, 9, 0$.
- If the given number has n digits, then the square root will contain $\frac{n}{2}$ or $\frac{n+1}{2}$ digits.
- Systematic computation of an exact square root requires the *Dvandvayoga* (Duplex) process.



6.1. Duplex Process (Dvandvayoga)

$$D(4) = 16 \quad (1)$$
$$D(43) = 24 \quad (2)$$
$$D(137) = 23 \quad (3)$$
$$D(1034) = 8 \quad (4)$$
$$D(10345) = 19 \quad (5)$$

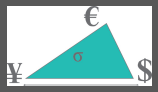
Got it?



6.2. Square Root of a Perfect Square

Find $\sqrt{1849}$.

Group in pairs, taking a single extra digit on the left as extra digit.



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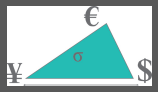
$$\begin{array}{r} \hline \quad 1 \quad 8 \;\text{—}\; 4 \quad 9 \\ 8) \qquad 2 \\ \hline \qquad 4 \end{array}$$



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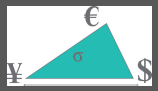
Group in pairs, taking a single extra digit on the left as extra digit.

$$\begin{array}{r} \hline \quad 1 \quad 8 \;\text{—}\; 4 \quad 9 \\ 8) \qquad 2 \\ \hline \qquad 4 \end{array}$$

4 is the largest integer whose square does not exceed 18.
$18/4$ is 4 with remainder 2.
The divisor 8 is two times 4.

Next we divide 24 by the divisor 8. This gives 3 remainder 0, placed as

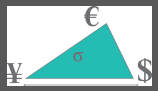
$$\begin{array}{r|l} & 1\ 8 - 4\ 9 \\ \hline 8) & \quad 2\ \ 0 \\ \hline & \quad 4\ \ 3 \end{array}$$



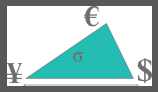
Next we divide 24 by the divisor 8. This gives 3 remainder 0, placed as

		1	8	—	4	9
$8)$			2		0	
			4		3	

Now we see 09 and we deduct from this the duplex of the last answer figure 3, i.e. $09 - D(3) = 09 - 3^2 = 09 - 9 = 0$. This means that the answer is exactly 43.



$$
\begin{array}{r}
1\ 3 - 6\ 9 \\
\hline
6) \quad 4 \\
\hline
3
\end{array}
$$



$$\begin{array}{r} 1\quad 3 - 6\quad 9 \\ 6)\quad\quad 4 \\ \hline 3 \end{array}$$

3 is the largest integer whose square does not exceed 13.
$13/3$ is 3 with remainder 4.
The divisor 6 is two times 3.
Next we divide 46 by the divisor 6. This gives 7 remainder 4, placed as

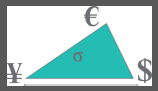
$$\begin{array}{r} 1\quad 3 - 6\quad\quad 9 \\ 6)\quad\quad 4\quad\quad 4 \\ \hline 3\quad\quad 7 \end{array}$$

$$\begin{array}{r|l} & 1\ \ 3\ \text{—}\ 6\ \ 9 \\ \hline 6) & \quad\ \ 4 \\ \hline & \quad\ \ 3 \end{array}$$

3 is the largest integer whose square does not exceed 13.
$13/3$ is 3 with remainder 4.
The divisor 6 is two times 3.
Next we divide 46 by the divisor 6. This gives 7 remainder 4, placed as

$$\begin{array}{r|l} & 1\ \ 3\ \text{—}\ 6\ \ \ 9 \\ \hline 6) & \quad\ \ 4\ \ \ 4 \\ \hline & \quad\ \ 3\ \ \ 7 \end{array}$$

$49 - D(7) = 0$, so 37 is the exact square root of 1369.



6.3. Exercises

Find the square root of the following.

a 3136

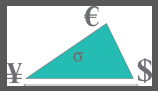
b 3969

c 5184

d 3721

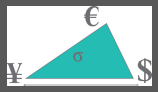
e 6889

f 1296



6.4. Larger Numbers

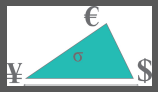
$$
\begin{array}{rcccccc}
 & 2 & 9 & 3 & 7 & 6 & 4 \\
\hline
10) & & 4 & & & & \\
\hline
 & & 5 & & . & &
\end{array}
$$



6.4. Larger Numbers

$$
\begin{array}{r}
2\;9\;3\;7\;6\;4 \\
\hline
10)\;\;4 \\
\hline
5\;\;\;\;\;\;\;.
\end{array}
$$

$$
\begin{array}{r}
2\;9\;3\;7\;6\;4 \\
\hline
10)\;\;4\;3 \\
\hline
5\;4\;\;\;\;\;.
\end{array}
$$

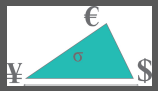


6.4. Larger Numbers

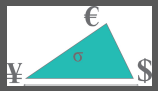
$$\begin{array}{r|l} & 2\ 9\ 3\ 7\ 6\ 4 \\ 10) & \quad 4 \\ \hline & 5\ \quad . \end{array}$$

$$\begin{array}{r|l} & 2\ 9\ 3\ 7\ 6\ 4 \\ 10) & \quad 4\ 3 \\ \hline & 5\ 4\ \quad . \end{array}$$

$$37 - D(4) = 37 - 16 = 21 = 2 \times 10 + 1.$$



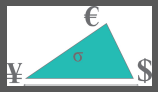
$$
\begin{array}{r}
2\ 9\ 3\ 7\ 6\ 4 \\
\hline
10)\ \ 4\ 3\ 1 \\
\hline
5\ 4\ 2.
\end{array}
$$



$$\begin{array}{rcccccc} & 2 & 9 & 3 & 7 & 6 & 4 \\ \hline 10) & & 4 & 3 & 1 & & \\ \hline & & 5 & 4 & 2. & & \end{array}$$

$$16 - D(42) = 16 - 16 = 0 = 0 \times 10 + 0.$$

$$\begin{array}{rcccccc} & 2 & 9 & 3 & 7 & 6 & 4 \\ \hline 10) & & 4 & 3 & 1 & 0 & \\ \hline & & 5 & 4 & 2. & 0 & \end{array}$$

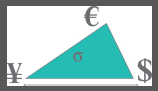


$$\begin{array}{r|rrrrrr} & 2 & 9 & 3 & 7 & 6 & 4 \\ \hline 10) & & 4 & 3 & 1 & & \\ \hline & & 5 & 4 & 2. & & \end{array}$$

$16 - D(42) = 16 - 16 = 0 = 0 \times 10 + 0.$

$$\begin{array}{r|rrrrrr} & 2 & 9 & 3 & 7 & 6 & 4 \\ \hline 10) & & 4 & 3 & 1 & 0 & \\ \hline & & 5 & 4 & 2. & 0 & \end{array}$$

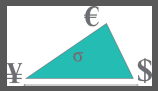
$4 - D(420) = 4 - 4 = 0.$ Complete.



6.5. General Square Roots

Find the first 5 figures of the square root of 38:

	3	8	.	0		0		0		0	0
12)			2		8		7		10		8
		6	.	1		6		4		4	



6.6. Exercises

Find the square root of the following.

a 119025

b 524176

c 519841

d 375769

7. Contact Information

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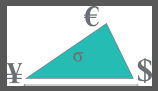

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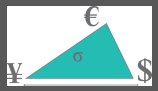
[3] Schonard, A. and Kokot, C. (2006). *Der Matheknüller*. http://www.matheknueller.de.

[4] Williams, K.R. (2002). *Vedic Mathematics - Teacher's Manual*. Advanced Level. Motilal Banarsidass Publishers Private Limited, Delhi. http://www.mlbd.com



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