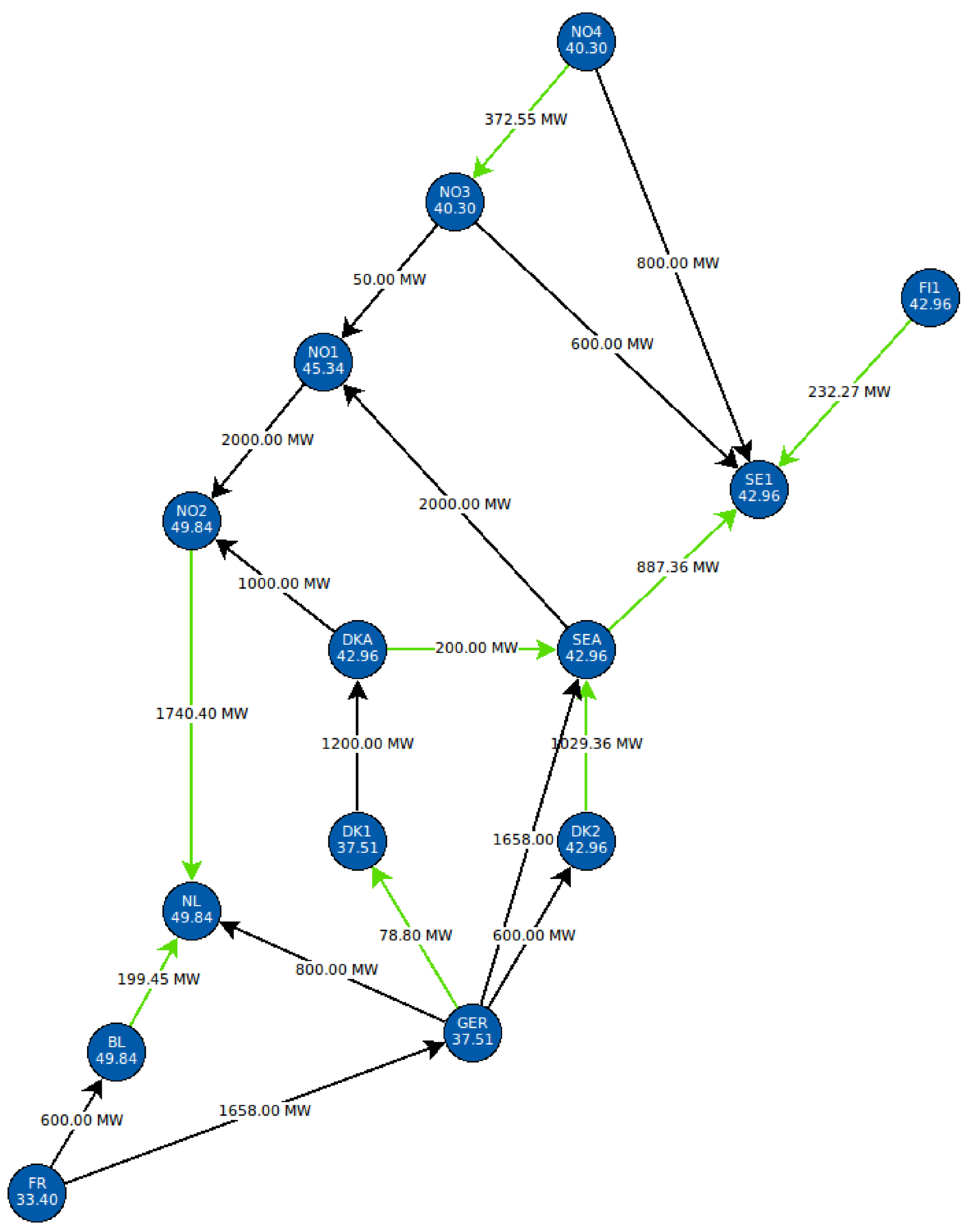
Clearing Coupled Day-Ahead Energy Markets

Johannes Müller, Alexander Martin, and Sebastian Pokutta
TU Darmstadt, Discrete Optimization



green: equal prices
black: tight transmission constraints

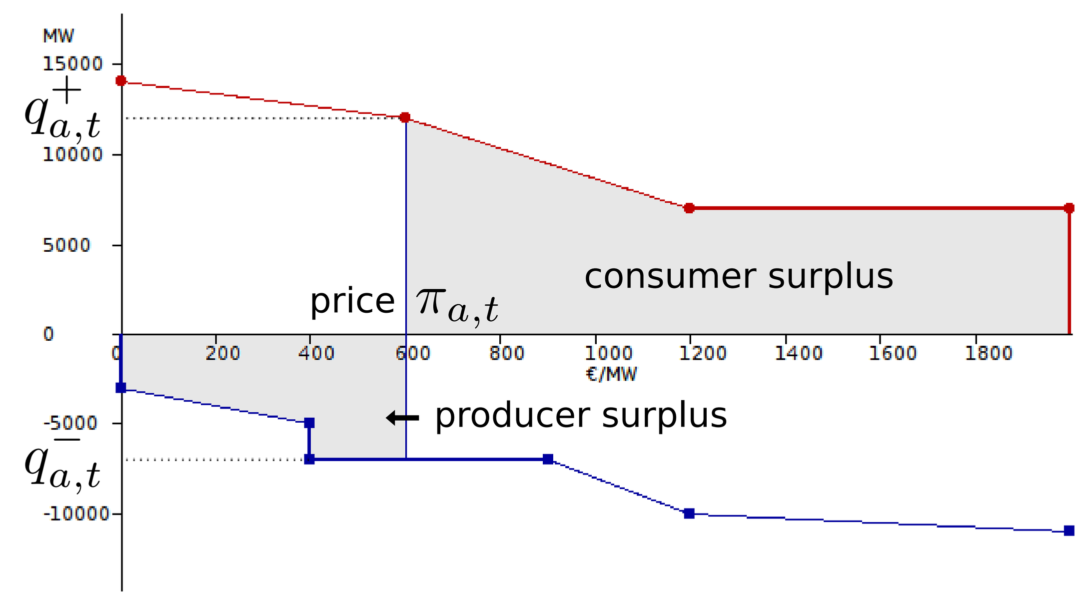
Introduction

Why do we need energy market coupling? Can't we just choose one price for all areas?

As the transmission **capacity is limited** it is not possible to balance the net demand of adjacent areas in general. Choosing one price in such a case, would lead to curtailment, for example a deficit in supply would occur at the chosen price. A better approach is, to allow for price differences and to use the transfer capacity to transmit energy from a low price area to a high price area.

How can the transmission capacity be used efficiently?

In contrast to classical market clearing via volume maximization, we **maximize the social welfare**. In other words, the consumer and producer surplus is maximized. This is not equivalent to volume maximization, as we face a market with combinatorial structure.



Why is it so complicated to determine efficient prices?

The auction system allows participants to buy or sell electricity for several hour slots with just one bid, the so called **block bid**. The price for this bid will be the average price of electricity weighted by the traded quantities. As block bids must satisfy the fill-or-kill condition, this imposes an additional combinatorial structure. This type of market is called a **smart market**, that has to be cleared with the help of combinatorial optimization. Unfortunately standard solvers cannot handle this challenge, so a special algorithm is needed.
The most difficult part is to **find the best block bid selection**.

What is the basic concept of the algorithm?

1. Find best integer solution of a relaxed MIQP-Model. Duality:
   - primal variables: transmission and executed quantities
   - dual variables: market clearing prices
2. If no block bid incurs a loss, we are done
3. Otherwise we add a new constraint to the model:
   - at least one block bid that incurred a loss, is not to be executed in the next run
   - go to phase 1.

We incorporated this technique into a branch-and-cut framework in order to compute clearing prices for the spot auction in about 7 seconds.

Economic Interpretation

There is a natural economic interpretation of optimal solutions: Given a fixed block bid selection, the complementary slackness of the Karush-Kuhn-Tucker conditions imply the following:

- prices are valid for the bid curves
- a price difference between adjacent areas implies that a transmission constraint is tight

Otherwise, it would be possible to transmit a higher quantity, generating a higher welfare, and thus the welfare would not be optimal.

Simplified model

Let β^* be a fixed block bid selection, then the model simplifies to.

$$\max \sum_{(a,t)\in A\times T} \left[\int_0^{q^+_{a,t}} \mathbf{p}^+_{a,t}(q)\,\mathrm{d}q - \int_{q^-_{a,t}}^0 \mathbf{p}^-_{a,t}(q)\,\mathrm{d}q \right]$$

$$s.t.\ \forall a \in A, t \in T$$

$$\text{quantity bounds:}\quad q^-_{a,t} \le 0 \le q^+_{a,t}$$

$$\text{net demand} = \text{net import:}\quad q^+_{a,t} + q^-_{a,t} + c_{a,t} = \sum_{c\in C_a^-} \tau_{c,t} - \sum_{c\in C_a^+} \tau_{c,t}$$

$$\forall c \in C, t \in T$$

$$\text{ATC:}\quad \underline{\tau}_{c,t} \le \tau_{c,t} \le \overline{\tau}_{c,t}$$

$$\text{ramp rate:}\quad -\tilde{\tau}_c \le \tau_{c,t} - \tau_{c,t-1} \le \tilde{\tau}_c$$

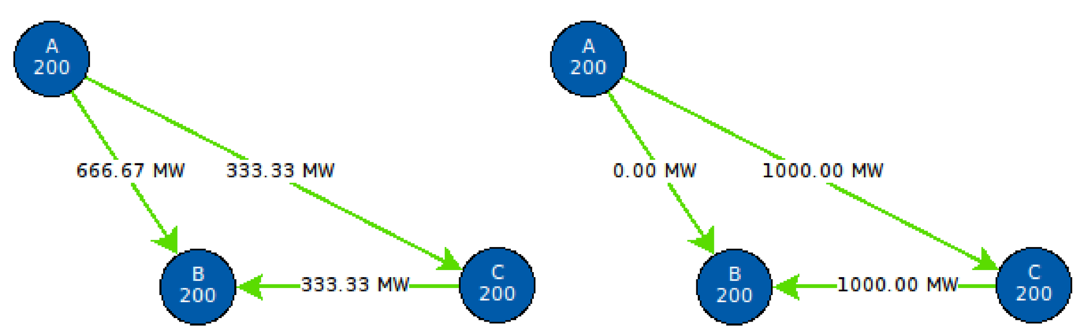
Notation:

$\tau_{c,t}$	transmission on connector c, hour t
$q^+_{a,t}$	bid curve demand in area a, hour t
$\mathbf{p}^+_{a,t}(q)$	utility of the consumption of q units
$\mathbf{p}^-_{a,t}(q)$	cost of the production of q units
$c_{a,t}$	net demand of block bids β^*
$c \in C^+_a$	connectors leaving area a

This model does **not contain price variables**. Prices will be calculated in a separate model with the subject to fulfill the block bid price conditions.

Uniqueness of the solution

For a fixed block bid selection there is one **unique optimal welfare** value. Though there may be non-unique transmission quantities, that generate the same welfare. We **minimize the squared transmissions**, to choose a unique flow.



Still there may be a range of welfare indifferent prices. A unique price is chosen by **minimizing the squared prices** without changing the welfare.